Generalized Threshold Factorization with Full Collinear Dynamics.

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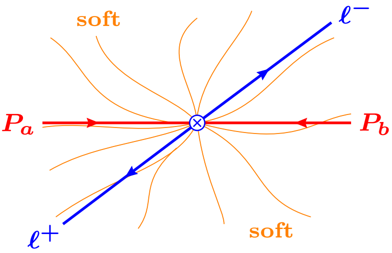
Motivation.

Drell-Yan production near threshold, $\tau \equiv Q^2/E_{\rm cm}^2 \to 1$:

$$\frac{d\sigma}{dQ} = \int dz\, \sigma_{ij}(z)\, [f_i \otimes f_j]\Big(\frac{\tau}{z}\Big)$$
$$= H_{ij}(Q) \int dk^0\, S(k^0)\, [f_i^{\rm thr} \otimes f_j^{\rm thr}]\Big(\tau + \frac{k^0}{E_{\rm cm}}\Big) \times \Big[1 + \mathcal{O}\big(1-\tau\big)\Big]$$

[Collins, Soper, Sterman '85-'88; Sterman '86]



- For steep PDFs, the integral is dominated by $z \sim 1$ even if $\tau \sim 10^{-4}$ at the LHC
- Useful approximation at partonic level: $\sigma_{ij} = H_{ij} \times S + \mathcal{O}[(1-z)^0]$
- Expansion in $1-z$ is key for N^{3}LO Higgs [Anastasiou et al. '14-'19]
- Recent progress in all-order understanding of next-to-leading power $\mathcal{O}[(1-z)^0]$ [Del Duca et al. '17] [Beneke et al. '18]

Motivation.

What if we measure rapidity Y in addition?

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}Y} = H_{ij}(Q) \int \mathrm{d}k^+ \mathrm{d}k^-\, S(k^+, k^-) \times f_i^{\mathrm{thr}}\Big(x_a + \frac{k^-}{E_{\mathrm{cm}}}\Big)\, f_i^{\mathrm{thr}}\Big(x_b + \frac{k^+}{E_{\mathrm{cm}}}\Big) \times \Big[1 + \mathcal{O}\big(1-\tau\big)\Big]$$

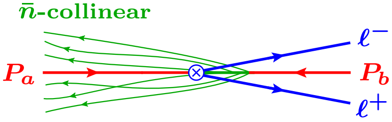
[Catani, Trentadue '89]

[Ahmed, Banerjee, Das, Dhani, Ravindran, Smith, van Neerven '07-'18; Owens, Westmark '17]

- Measurement sets momentum fractions $x_{a,b} = \frac{Q}{E_{\mathrm{cm}}} e^{\pm Y}$
- $\tau = x_a x_b \to 1$ assumes $x_a \to 1$ and $x_b \to 1$

QUESTION: What happens if we relax one of these assumptions?
What is the physical interpretation of that?

Factorization at collinear endpoint.



- $x_a \to 1$ means $Y \to Y_{\max} \equiv \ln \frac{E_{\rm cm}}{Q}$
- Let $\lambda^2 \sim 1 - \frac{q^-}{E_{\rm cm}} \sim 1 - x_a \ll 1$
- Keep q^+ and x_b generic

- Hadronic final state X becomes $\bar{n}$-collinear near endpoint

$$p_X^\mu = (P_a^- - q^-, P_b^+ - q^+, p_{X,\perp}) \sim Q(\lambda^2, 1, \lambda)$$

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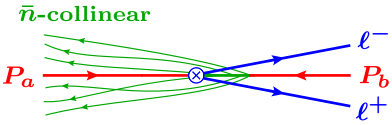

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$$p_X^\mu = (P_a^- - q^-, P_b^+ - q^+, p_{X,\perp}) \sim Q(\lambda^2, 1, \lambda)$$

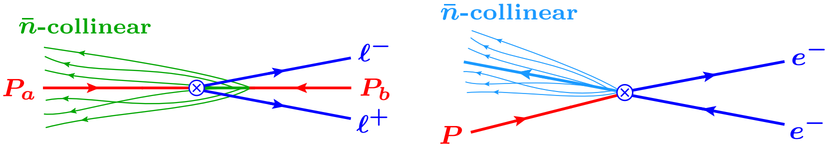
$\Rightarrow$ Resulting factorization theorem at leading power in λ:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q^+\mathrm{d}q^-} = H_{ij}(q^+q^-, \mu) \int \mathrm{d}t\, B_j\Big(t, \frac{q^+}{E_{\rm cm}}, \mu\Big)\, f_i^{\rm thr}\Big(\frac{q^-}{E_{\rm cm}} + \frac{t}{q^+q^-}, \mu\Big)$$

- Key step: Power counting in overall momentum conservation

$$\delta\Big[\underbrace{(\omega_a^- - q^-)}_{\mathcal{O}(\lambda^2)} + \underbrace{k_b^-}_{\mathcal{O}(\lambda^2)}\Big]\,\delta\Big[\underbrace{(\omega_b^+ - q^+)}_{\mathcal{O}(1)} + \underbrace{\cancel{k_a^+}}_{\mathcal{O}(\lambda^2)}\Big]$$

Connection to endpoint DIS.



- Modes, anom. dims. & convolution structure are the same as for endpoint DIS
- $x_a \sim q^- / E_{\rm cm} \to 1$ takes the role of $x_{\rm Bjorken} \to 1$:

$$\frac{d\sigma_{\rm DY}}{dq^+ dq^-} = H_{ij}(q^+q^-, \mu) \int dt\, B_j\Big(t, \frac{q^+}{E_{\rm cm}}, \mu\Big) \; f_i^{\rm thr}\Big(\frac{q^-}{E_{\rm cm}} + \frac{t}{q^+q^-}, \mu\Big)$$

$$\frac{d\sigma_{\rm DIS}}{dx_B} = H_{ij}(Q^2, \mu) \int ds\, J_j(s, \mu) \; f_i^{\rm thr}\Big(x_B + \frac{s}{Q^2}, \mu\Big)$$

- Second, unconstrained Bjorken fraction $x_b \sim q^+ / E_{\rm cm}$ is beam function argument

Measuring q_T in addition.

- Only $\bar{n}$-collinear radiation contributes recoil for $q_T \gtrsim \lambda Q$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q^+ \mathrm{d}q^- \mathrm{d}\vec{q}_T} = H_{ij} \int \mathrm{d}t \, B_j\Big(t, \frac{q^+}{E_\mathrm{cm}}, \vec{q}_T\Big) \, f_i^\mathrm{thr}\Big(\frac{q^-}{E_\mathrm{cm}} + \frac{t}{q^+ q^-}\Big)$$

  - Same double-differential $\mathrm{SCET_I}$ beam function as in $(q_T, \mathcal{T}_0)$ resummation
    [Jain, Procura, Waalewijn, Zeune ’11-’14; Lustermans, JM, Waalewijn, Tackmann ’19]

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- Change variables from (q^+, q^-) back to $(Q, Y) \leftrightarrow (x_a, x_b)$:

$$x_{a,b} = \frac{Q}{E_\mathrm{cm}} e^{\pm Y} \quad \neq \quad \frac{q^\pm}{E_\mathrm{cm}} = \frac{\sqrt{Q^2 + q_T^2}}{E_\mathrm{cm}} e^{\pm Y}$$

- Power-counting parameter is now $\lambda^2 \sim 1 - x_a$. Reexpand:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_a \mathrm{d}x_b\, \mathrm{d}\vec{q}_T} = H_{ij} \int \mathrm{d}t\, B_j(t, x_b, \vec{q}_T)\, f_i^\mathrm{thr}\Big(x_a + \frac{q_T^2}{2Q^2} + \frac{t}{Q^2}\Big)$$

- What happened here? Look at $1 -$ PDF argument $\sim \lambda^2$:

$$\Big(1 - \frac{\sqrt{Q^2 + q_T^2}\, e^Y}{E_\mathrm{cm}}\Big) - \frac{t}{Q^2 + q_T^2} = (1 - x_a) - \frac{q_T^2}{2Q^2} - \frac{t}{Q^2} + \mathcal{O}(\lambda^4)$$

Back to the inclusive spectrum.

- Start from the triple-differential spectrum:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_a \mathrm{d}x_b\, \mathrm{d}\vec{q}_T} = H_{ij} \int \mathrm{d}t\, B_j(t, x_b, \vec{q}_T)\, f_i^{\mathrm{thr}}\Big(x_a + \frac{q_T^2}{2Q^2} + \frac{t}{Q^2}\Big)$$

Integrate over $\vec{q}_T$, shift $t' \equiv t + \frac{q_T^2}{2}$ $\Rightarrow$ inclusive factorization theorem for (Q, Y):

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_a \mathrm{d}x_b} = H_{ij} \int \mathrm{d}t'\, B'_j(t', x_b)\, f_i^{\mathrm{thr}}\Big(x_a + \frac{t'}{Q^2}\Big)$$

- Same form as $\mathrm{d}\sigma/\mathrm{d}q^+\mathrm{d}q^-$, but with a new $\mathrm{SCET_I}$ beam function:

$$B'_j(t', x) \equiv \int \mathrm{d}^2\vec{k}_T\, B_j\Big(t' - \frac{k_T^2}{2}, \vec{k}_T, x\Big)$$

  - Identical RGE as $B_j(t, x)$, but different constant terms
  - Calculated matching coefficient $\mathcal{I}'_{qk}(t', z)$ through $\mathcal{O}(\alpha_s^2)$ by projecting $\mathcal{I}_{qk}(t, z, \vec{k}_T)$ onto t' [two-loop inputs: Gaunt, Stahlhofen '14]

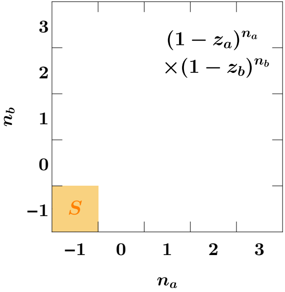
Power counting in the partonic cross section.

- Parametrize partonic cross section as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_a \mathrm{d}x_b} = \int \frac{\mathrm{d}z_a}{z_a} \frac{\mathrm{d}z_b}{z_b} \, \sigma_{ij}(z_a, z_b) \, f_i\Big(\frac{x_a}{z_a}\Big) \, f_j\Big(\frac{x_b}{z_b}\Big)$$

- Soft threshold factorization only predicts terms

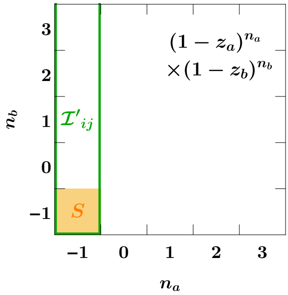
$H_{q\bar{q}} \, S \sim \dfrac{1}{1 - z_a} \dfrac{1}{1 - z_b}$ in the $q\bar{q}$ channel



Power counting in the partonic cross section.

- Parametrize partonic cross section as

$$\frac{d\sigma}{dx_a dx_b} = \int \frac{dz_a}{z_a}\frac{dz_b}{z_b}\,\sigma_{ij}(z_a, z_b)\, f_i\Big(\frac{x_a}{z_a}\Big)\, f_j\Big(\frac{x_b}{z_b}\Big)$$



- Soft threshold factorization only predicts terms

$$H_{q\bar q}\, S \sim \frac{1}{1 - z_a}\,\frac{1}{1 - z_b}$$ in the $q\bar q$ channel

- Collinear endpoint factorization predicts all terms $\sim \dfrac{F(z_b)}{1 - z_a}$

$$\sigma_{qj}(z_a, z_b) = H_{q\bar q}(Q^2) \times \mathcal{I}'_{qj}\big[Q^2(1 - z_a), z_b\big] + \mathcal{O}[(1 - z_a)^0]$$

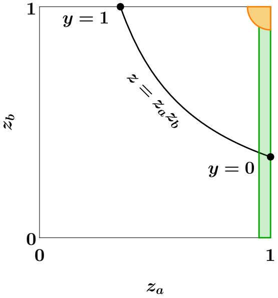
- Corollary: Soft function captures singular terms within the beam function

$$\mathcal{I}'_{ij}(\omega k^+, z) = \delta_{ij}\, S\big[\omega(1 - z), k^+\big] + \mathcal{O}[(1 - z)^0]$$

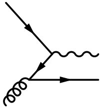
✔ Checked through $\mathcal{O}(\alpha_s^2)$

Analytic NLO check: qg

- NNLO Drell-Yan rapidity spectrum is known analytically
  [Anastasiou, Dixon, Melnikov, Petriello ’02-’03]
  - ▶ Parametrized in terms of $z = z_a z_b$ and $y \in [0, 1]$
  - ✔ Analytically expand NLO results as $z_a \to 1$ with z_b generic → full agreement



- Instructive to look at some NLO terms explicitly:



$$= \sigma_B\, T_F \Big\{ \delta(y) \Big[2P_{qg}(z) \ln \frac{(1-z)^2}{z} + 4z(1-z) \Big] + 2\, P_{qg}(z) \mathcal{L}_0(y) + 4z(1-z) + 2(1-z)^2 y \Big\}$$

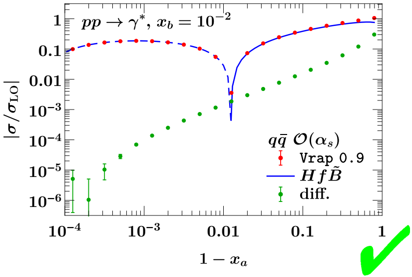
- Most nontrivial term, with $\mathcal{L}_0(x) \equiv \Big[\frac{1}{x}\Big]_+$:

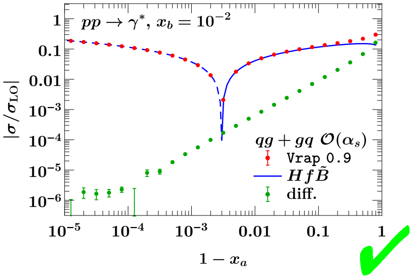
$$P_{qg}(z)\, \mathcal{L}_0(y)\, \mathrm{d}z\, \mathrm{d}y$$

$$= P_{qg}(z_b) \Big\{ \mathcal{L}_0(1-z_a) + \delta(1-z_a) \Big[\ln \frac{2z_b}{1+z_b} - \ln(1-z_b) \Big] \Big\} \mathrm{d}z_a \mathrm{d}z_b$$

$$+ \mathcal{O}[(1-z_a)^0]$$

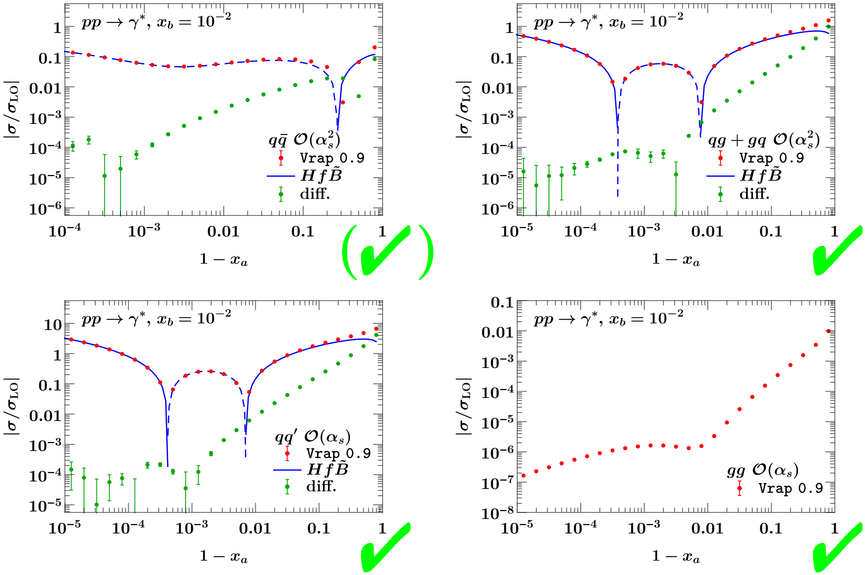
Missing in $\mathcal{I}$, but captured by $\mathcal{I}'$ ...





[**Vrap 0.9**: Anastasiou, Dixon, Melnikov, Petriello ’03]

Numerical NNLO check.



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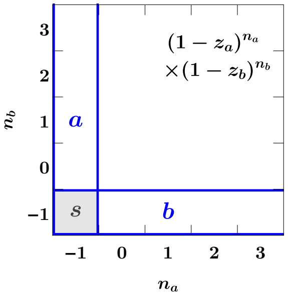
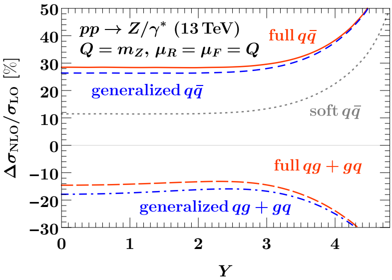
Generalized Threshold Approximation.

So we're done factorizing. What next?

- Let's combine all our leading-power knowledge of the fixed-order cross section:

$$\sigma_{ij} = \sigma^{\mathrm{a}}_{ij} + \sigma^{\mathrm{b}}_{ij} - \sigma^{\mathrm{s}}_{ij} + \mathcal{O}(1)$$





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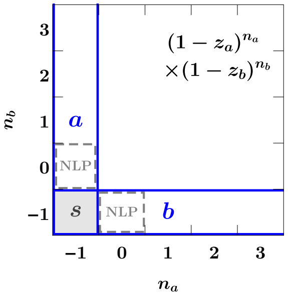
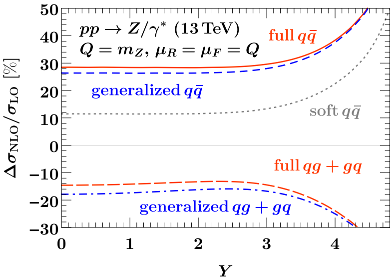
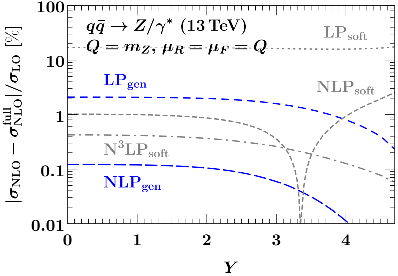
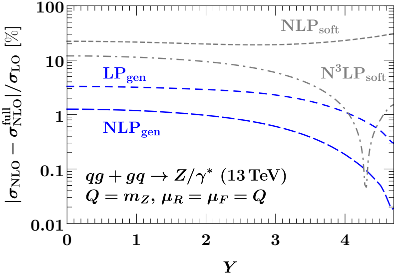


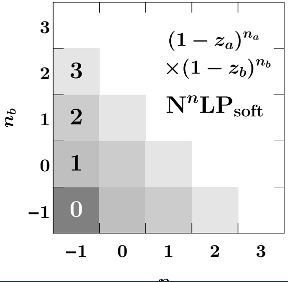


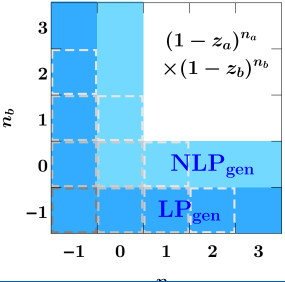
- Leading-power generalized threshold contains full soft NLP at fixed order:

$$\int \mathrm{d}z_a \mathrm{d}z_b\, \delta(z - z_a z_b)\, (1-z_a)^{n_a} (1-z_b)^{n_b} \sim (1-z)^{n_a+n_b+1}$$



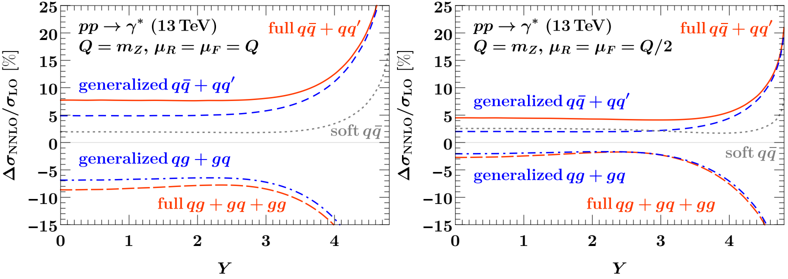






- Generalized threshold expansion converges faster for all Y
- For qg channel, $\mathrm{LP_{gen}}$ already better than $\mathrm{N^3LP_{soft}}$

NNLO approximants.



→ $\mathrm{LP}_{\mathbf{gen}}$ again closely tracks full

Summary.

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Generalized threshold factorization for LHC rapidity spectra:

- Extend soft factorization to full collinear dynamics at endpoint
  - Weakest known limit to have virtuals factorize in inclusive spectra
  - Offdiagonal partonic channels are captured at leading power
- Obtained & checked new beam functions through NNLO
- First application: Fixed-order approximants for Drell-Yan spectra
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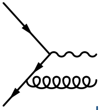
Thank you for your attention!

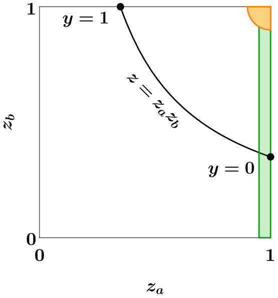
...invitation for discussion on the next slide ...

Invitation for discussion.

Analytic NLO check: $q\bar{q}$



$$= \sigma_B\, C_F \Big\{ \big[\delta(y) + \delta(1-y)\big] \Big[\delta(1-z)(4\zeta_2 - 8) + 8(1+z^2)\mathcal{L}_1(1-z) - 2\frac{1+z^2}{1-z}\ln z + 2 - 2z\Big] + 2(1+z^2)\, \mathcal{L}_0(1-z)[\mathcal{L}_0(y) + \mathcal{L}_0(1-y)] - 2(1-z) \Big\}$$

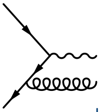


- Most nontrivial term, with $\mathcal{L}_0(x) \equiv \left[\frac{1}{x}\right]_+$

$$\mathcal{L}_0(1-z)[\mathcal{L}_0(y) + \mathcal{L}_0(1-y)]\, \mathrm{d}z\, \mathrm{d}y$$

$$= \Big[\frac{\pi^2}{6}\delta(1-z_a)\delta(1-z_b) - \mathcal{L}_1(1-z_a)\delta(1-z_b) + \mathcal{L}_0(1-z_a)\mathcal{L}_0(1-z_b) - \delta(1-z_a)\mathcal{L}_1(1-z_b) + \delta(1-z_a)\frac{\ln\frac{2z_b}{1+z_b}}{1-z_b}\Big]\mathrm{d}z_a \mathrm{d}z_b + \mathcal{O}[(1-z_a)^0]$$

Analytic NLO check: $q\bar{q}$



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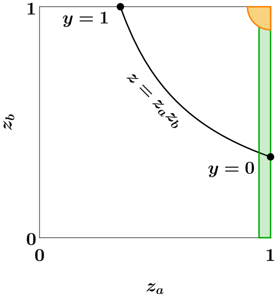

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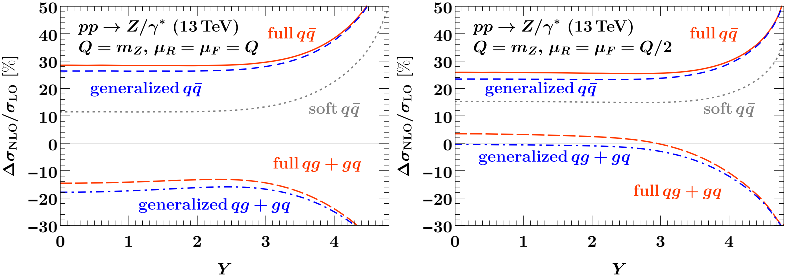
- Several soft threshold factorizations for the rapidity spectrum neglect this term, and conclude $\sigma_{ij}(z,y) = [\delta(y) + \delta(1-y)]\,\sigma_{ij}^{\mathrm{soft}}(z) + \mathcal{O}(1)$
  [Bolzoni '06; Mukherjee, Vogelsang '06; Becher, Neubert, Xu '07; Bonvini, Forte, Ridolfi '10]

✗ Our results indicate that this misses leading-power soft terms already at LL.

Backup.

NLO approximants (different scales, channels)



NLO subleading power (different scales)