

NNLO QCD predictions for the LHC with antenna subtraction

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Les Houches Workshop, Physics at the TeV colliders
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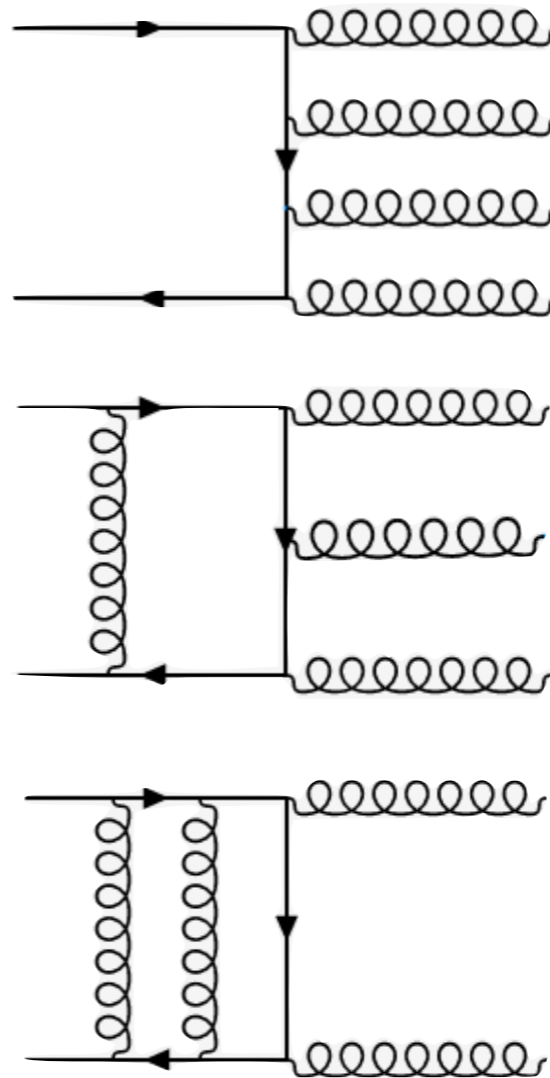
Max-Planck-Institut für Physik
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Anatomy of an NNLO calculation

$$\begin{aligned}
 d\hat{\sigma}_{NNLO} &= \int_{d\Phi_4} d\hat{\sigma}_{NNLO}^{RR} \\
 &+ \int_{d\Phi_3} d\hat{\sigma}_{NNLO}^{RV} \\
 &+ \int_{d\Phi_2} d\hat{\sigma}_{NNLO}^{VV}
 \end{aligned}$$



- double-unresolved
- single-unresolved
- single-unresolved
- $1/\epsilon^2$; $1/\epsilon$
- $1/\epsilon^4$; $1/\epsilon^3$;
 $1/\epsilon^2$; $1/\epsilon$;

Assume all matrix elements are available

- Tree level matrix elements (RR) $2 \rightarrow n+2$
 - One-loop matrix elements (RV) $2 \rightarrow n+1$
 - Two loop matrix elements (VV) $2 \rightarrow n$
- } Form NLO correction to $2 \rightarrow n+1$

Infrared singularities: real radiation

NLO

- single collinear: $p_a // p_b$
- single soft: $E_a \rightarrow 0$

NNLO

- Triple collinear: $p_a // p_b // p_c$
- Double single collinear: $p_a // p_b ; p_c // p_d$
- Soft/collinear: $E_a \rightarrow 0 , p_b // p_c$
- Double soft: $E_a \rightarrow 0 , E_b \rightarrow 0$
- One-loop virtual correction to NLO singularities

NNLO antenna subtraction

$$\begin{aligned}
 d\hat{\sigma}_{NNLO} &= \int_{d\Phi_4} \left(d\hat{\sigma}_{NNLO}^{RR} - d\hat{\sigma}_{NNLO}^S \right) \\
 &+ \int_{d\Phi_3} \left(d\hat{\sigma}_{NNLO}^{RV} - d\hat{\sigma}_{NNLO}^T \right) \\
 &+ \int_{d\Phi_2} \left(d\hat{\sigma}_{NNLO}^{VV} - d\hat{\sigma}_{NNLO}^U \right)
 \end{aligned}$$

$$d\hat{\sigma}_{NNLO}^S \quad d\hat{\sigma}_{NNLO}^T$$

- mimic RR,RV in unresolved limits

$$d\hat{\sigma}_{NNLO}^T \quad d\hat{\sigma}_{NNLO}^U$$

- analytically cancel the poles in RV and VV matrix elements

- NNLO cross section with each line **finite** and **integrable** in d=4 dimensions

Implementation in parton-level event generator

- Generate particle momenta for (n), (n+1), (n+2)
- Reconstruct observable
- Weight with squared matrix elements
- Subtract/add real radiation singularities

Colour ordering

- QCD amplitudes in colour basis

All gluon

$$\mathcal{A}_n^{\text{tree}}(\{k_i, \lambda_i, a_i\}) = g^{n-2} \sum_{\sigma \in S_n / Z_n} \text{Tr}(T^{a_{\sigma(1)}} \dots T^{a_{\sigma(n)}}) A_n^{\text{tree}}(\sigma(1^{\lambda_1}), \dots, \sigma(n^{\lambda_n}))$$

Quark pair plus gluons

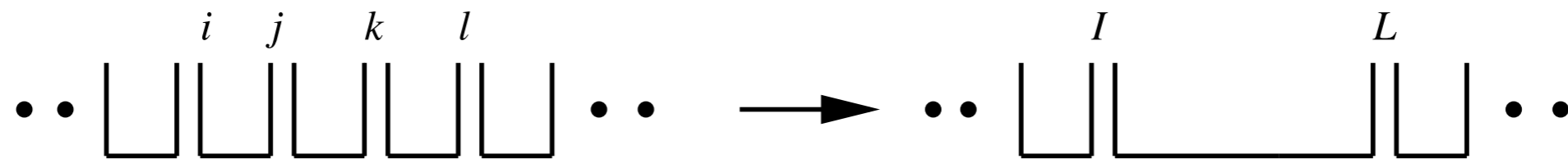
$$\mathcal{A}_n^{\text{tree}} = g^{n-2} \sum_{\sigma \in S_{n-2}} (T^{a_{\sigma(3)}} \dots T^{a_{\sigma(n)}})_{i_2}^{\bar{j}_1} A_n^{\text{tree}}(1_{\bar{q}}^{\lambda_1}, 2_q^{\lambda_2}, \sigma(3^{\lambda_3}), \dots, \sigma(n^{\lambda_n}))$$

- Real radiation infrared singularities only between colour adjacent partons
- Well defined patterns from colour-connections

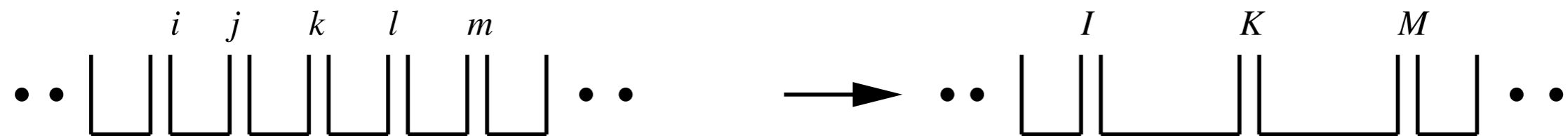
Antenna subtraction at NNLO (RR)

$$d\sigma_{NNLO}^S = d\sigma_{NNLO}^{S,a} + d\sigma_{NNLO}^{S,b} + d\sigma_{NNLO}^{S,c} + d\sigma_{NNLO}^{S,d} + d\sigma_{NNLO}^{S,e}$$

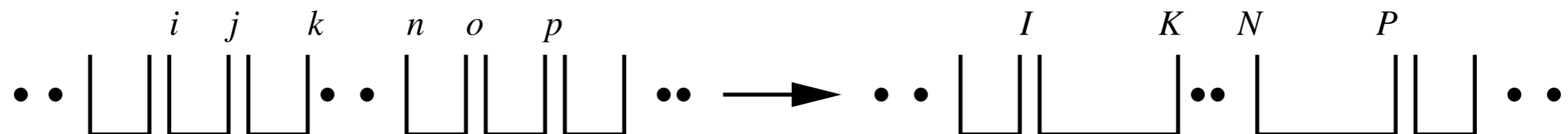
- (a) one unresolved parton \rightarrow three parton antenna function X_{ijk}
- (b) two colour-connected unresolved partons \rightarrow four parton antenna function X_{ijkl}



- (c) two almost colour connected unresolved partons \rightarrow strongly ordered product of non-independent three parton antenna functions $X_{ijk}X_{klm}, X_{klm}X_{ijk}$

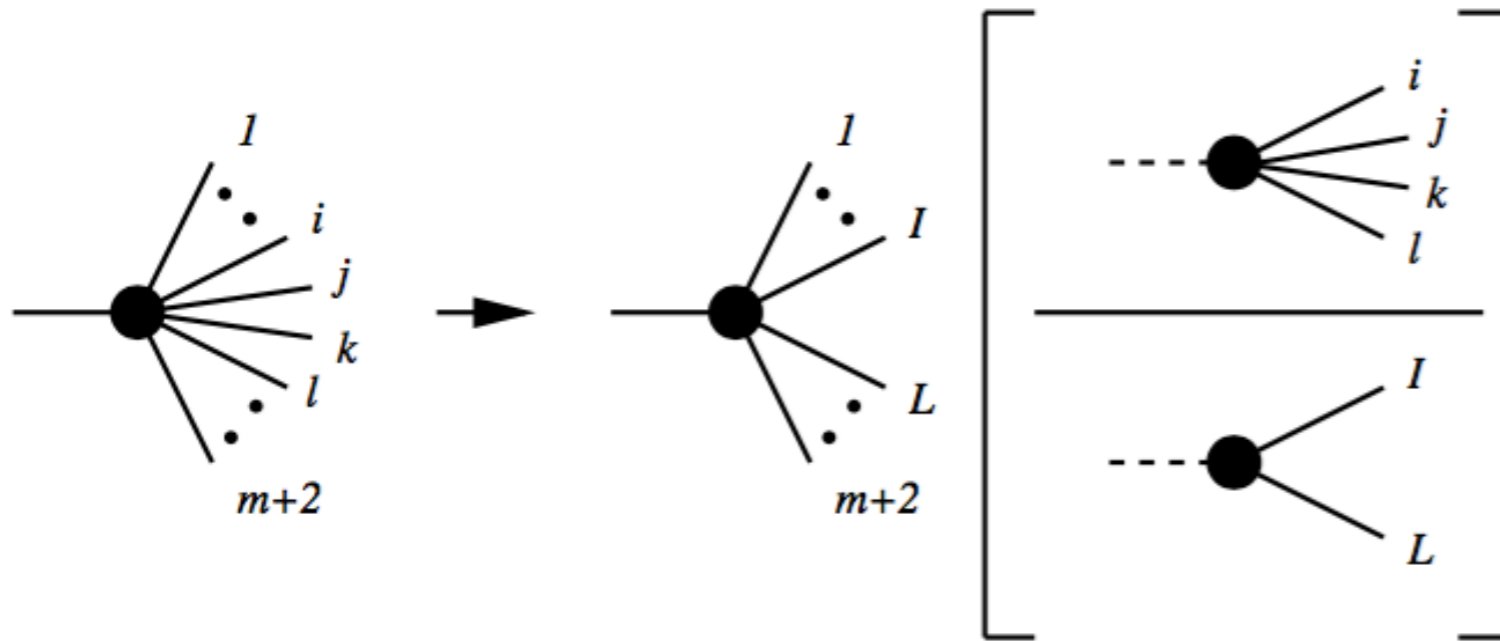


- (d) two colour unconnected unresolved partons \rightarrow product of independent three parton antenna functions $X_{ijk}X_{nop}$



- (e) subtracts large angle soft radiation \rightarrow soft factor S_{ajc}

Two colour connected partons



Four-parton
antenna function

$$\begin{aligned}
 d\sigma_{NNLO}^{S,b} = & \mathcal{N} \sum_{m+2} d\Phi_{m+2}(p_1, \dots, p_{m+2}; q) \frac{1}{S_{m+2}} \\
 & \times \left[\sum_{jk} (X_{ijkl}^0 - X_{ijk}^0 X_{IKl}^0 - X_{jkl}^0 X_{iJL}^0) \right. \\
 & \left. \times |\mathcal{M}_m(p_1, \dots, \tilde{p}_I, \tilde{p}_L, \dots, p_{m+2})|^2 J_m^{(m)}(p_1, \dots, \tilde{p}_I, \tilde{p}_L, \dots, p_{m+2}) \right],
 \end{aligned}$$

$$A_4^0(i_q, j_g, k_g, l_{\bar{q}})$$

smoothly interpolates
colour connected
unresolved limits

$$E_j, E_k \rightarrow 0$$

$$S_{ijkl}$$

$$p_i // p_j // p_k$$

$$P_{qgg \rightarrow Q}(w, x, y)$$

$$E_k \rightarrow 0, p_i // p_j$$

$$S_{q;gg\bar{q}} P_{qg \rightarrow Q}(z)$$

$$p_i // p_j, p_k // p_l$$

$$P_{qg \rightarrow Q}(z) P_{\bar{q}g \rightarrow \bar{Q}}(y)$$

- Phase space mapping $(i,j,k,l) \rightarrow (I,L)$

$$p_I = xp_i + r_1 p_j + r_2 p_k + zp_l$$

$$p_L = (1-x)p_i + (1-r_1)p_j + (1-r_2)p_k + (1-z)p_l$$

$$x = \frac{1}{2(s_{12} + s_{13} + s_{14})} \left[(1 + \rho) s_{1234} - r_1 (s_{23} + 2s_{24}) - r_2 (s_{23} + 2s_{34}) + (r_1 - r_2) \frac{s_{12}s_{34} - s_{13}s_{24}}{s_{14}} \right]$$

$$z = \frac{1}{2(s_{14} + s_{24} + s_{34})} \left[(1 - \rho) s_{1234} - r_1 (s_{23} + 2s_{12}) - r_2 (s_{23} + 2s_{13}) - (r_1 - r_2) \frac{s_{12}s_{34} - s_{13}s_{24}}{s_{14}} \right]$$

$$\rho = \left[1 + \frac{(r_1 - r_2)^2}{s_{14}^2 s_{1234}^2} \lambda(s_{12} s_{34}, s_{14} s_{23}, s_{13} s_{24}) + \frac{1}{s_{14} s_{1234}} \left\{ 2(r_1(1-r_2) + r_2(1-r_1))(s_{12}s_{34} + s_{13}s_{24} - s_{23}s_{14}) + 4r_1(1-r_1)s_{12}s_{24} + 4r_2(1-r_2)s_{13}s_{34} \right\} \right]^{\frac{1}{2}},$$

$$r_1 = \frac{s_{23} + s_{24}}{s_{12} + s_{23} + s_{24}}$$

$$r_2 = \frac{s_{34}}{s_{13} + s_{23} + s_{34}}$$

$$d\Phi_{m+2}(p_1, \dots, p_{m+2}) = d\Phi_m(p_1, \dots, p_I, p_L, \dots, p_{m+2}) \cdot d\Phi_{X_{ijkl}}(p_i, p_j, p_k, p_l; p_I, p_L)$$

$$\{p_i, p_j, p_k, p_l\} \rightarrow \{p_I, p_L\}$$

smoothly interpolates
colour connected
unresolved limits

$$E_j, E_k \rightarrow 0$$

$$p_I = p_i$$
$$p_L = p_l$$

$$p_i // p_j // p_k$$

$$p_I = p_i + p_j + p_k$$
$$p_L = p_l$$

$$E_k \rightarrow 0, p_i // p_j$$

$$p_I = p_i + p_j$$
$$p_L = p_l$$

$$p_i // p_j, p_k // p_l$$

$$p_I = p_i + p_j$$
$$p_L = p_k + p_l$$

Antenna functions and types

- all antennae can be derived from physical matrix elements in QCD
- colour ordered pair of hard partons (radiators) with radiation in between
 - hard quark-antiquark pair
 - hard quark-gluon pair
 - hard gluon-gluon pair
- three parton antennae → one unresolved parton
- four-parton antennae → two unresolved partons
- can be at tree level or one loop
- can be massless or massive
- all have three antenna types
 - final-final antenna
 - initial-final antenna
 - initial-initial antenna

$$X_3^0(i, j, k)$$

$$X_4^0(i, j, k, l)$$

$$X_3^1(i, j, k)$$

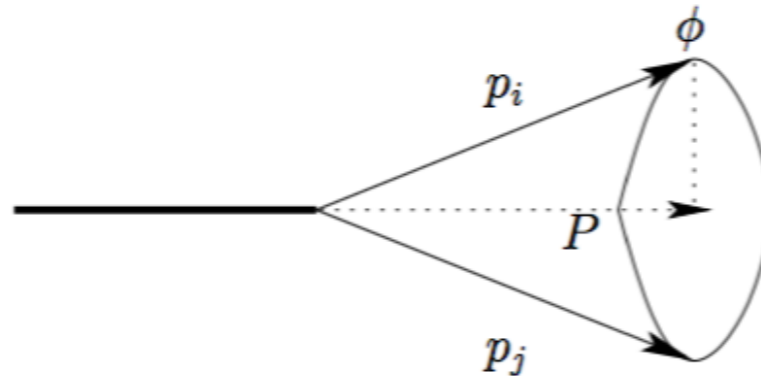
Angular averaging

- **Antenna** functions are **scalar objects** → do not subtract **angular correlations** in gluon splitting
- Angular correlations **vanish** after **integration** over the **azimuthal angle**

$$\frac{1}{2\pi} \int_0^{2\pi} d\phi (p_l \cdot k_\perp) = 0, \quad \frac{1}{2\pi} \int_0^{2\pi} d\phi (p_l \cdot k_\perp)^2 = -k_\perp^2 \frac{p \cdot p_l n \cdot p_l}{p \cdot n}$$

$$\Theta_{F_3^0}(i, j, z, k_\perp) \sim A \cos(2\phi + \alpha)$$

- Make **fully local** subtraction by **combining** phase space points related to each other by a 90 degree **rotation** of the system of unresolved partons



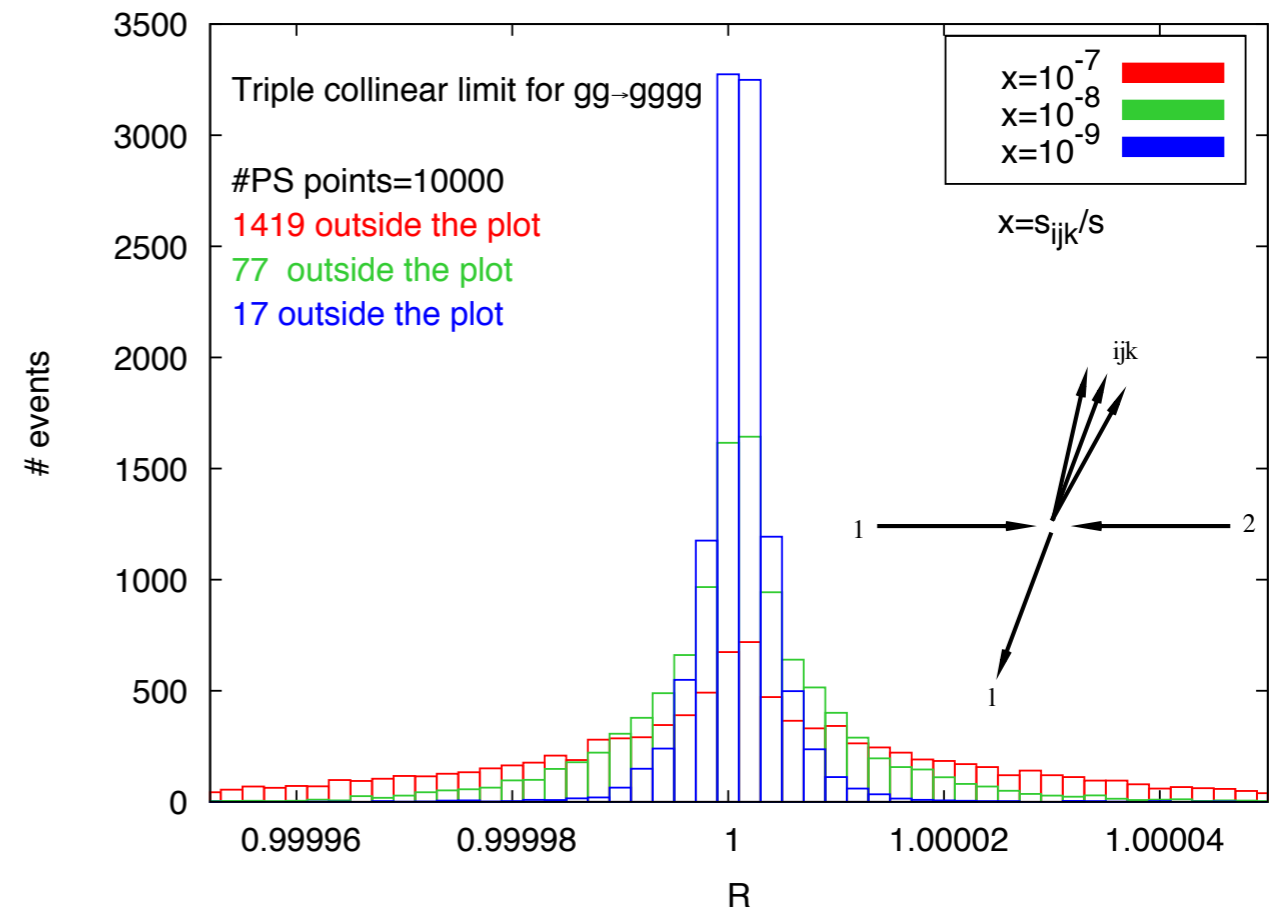
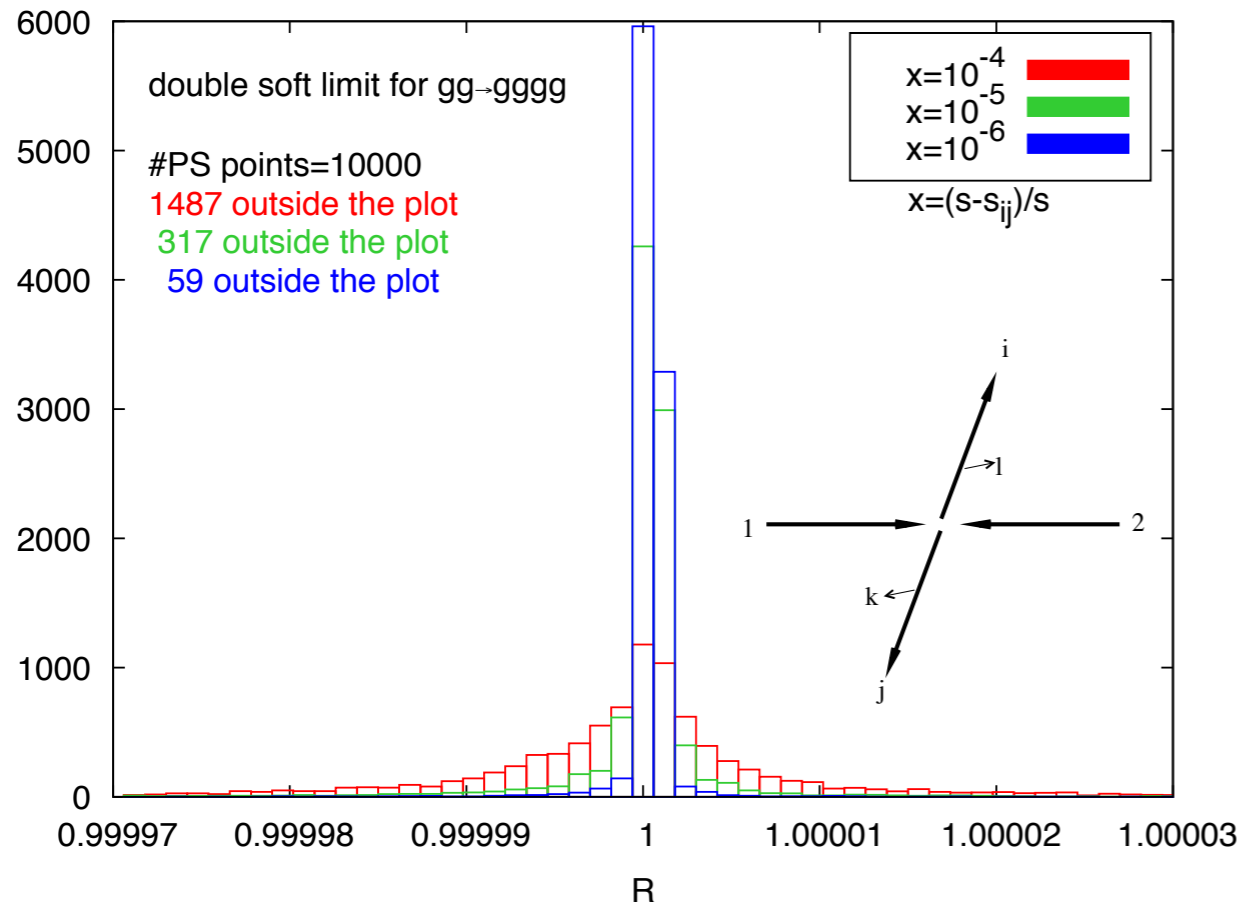
$$p_i^\mu = zp^\mu + k_\perp^\mu - \frac{k_\perp^2}{z} \frac{n^\mu}{2p \cdot n},$$

$$\text{with } 2p_i \cdot p_j = -\frac{k_\perp^2}{z(1-z)},$$

$$p_j^\mu = (1-z)p^\mu - k_\perp^\mu - \frac{k_\perp^2}{1-z} \frac{n^\mu}{2p \cdot n},$$

$$p^2 = n^2 = k_\perp \cdot p = k_\perp \cdot n = 0$$

Antenna subtraction at work



Double unresolved emission

- Generate phase space trajectories that approach singular region of the phase space
- Infrared behaviour of subtraction term mimics the behaviour of the matrix element

$$R = \frac{d\sigma_{NNLO}^R}{d\sigma_{NNLO}^S} \xrightarrow{l_g, k_g \rightarrow 0} 1$$

Double virtual antenna contribution

	a	b	b, c	d	e
$d\hat{\sigma}_{NNLO}^S$	$X_3^0 \mathcal{M}_{m+3}^0 ^2$	$X_4^0 \mathcal{M}_{m+2}^0 ^2$	$X_3^0 X_3^0 \mathcal{M}_{m+2}^0 ^2$	$X_3^0 X_3^0 \mathcal{M}_{m+2}^0 ^2$	$S X_3^0 \mathcal{M}_{m+2}^0 ^2$
$\int_1 d\hat{\sigma}_{NNLO}^{S,1}$	$\mathcal{X}_3^0 \mathcal{M}_{m+3}^0 ^2$	–	$\mathcal{X}_3^0 X_3^0 \mathcal{M}_{m+2}^0 ^2$	–	$S X_3^0 \mathcal{M}_{m+2}^0 ^2$
$\int_2 d\hat{\sigma}_{NNLO}^{S,2}$	–	$\mathcal{X}_4^0 \mathcal{M}_{m+2}^0 ^2$	–	$\mathcal{X}_3^0 \mathcal{X}_3^0 \mathcal{M}_{m+2}^0 ^2$	–

- Integrated **double unresolved** emission of RR process \propto tree level **double soft** function
- Integrated **iterated** NLO emissions of RR process
- Integrated **single unresolved** emission from RV process \propto tree level **single soft** function
- Integrated **single unresolved** emission of RV process \propto one loop **single soft** function

	$d\hat{\sigma}_{NNLO}^T$		
Final State Particles	a	a	(b, c)
$m + 1$	$X_3^1 \mathcal{M}_{m+2}^0 ^2$	$X_3^0 \mathcal{M}_{m+2}^1 ^2$	$\mathcal{X}_3^0 X_3^0 \mathcal{M}_{m+2}^0 ^2$
m	$\mathcal{X}_3^1 \mathcal{M}_{m+2}^0 ^2$	$\mathcal{X}_3^0 \mathcal{M}_{m+2}^1 ^2$	$\mathcal{X}_3^0 \mathcal{X}_3^0 \mathcal{M}_{m+2}^0 ^2$

Double virtual antenna contribution

	<i>a</i>	<i>b</i>	<i>b, c</i>	<i>d</i>	<i>e</i>
$d\hat{\sigma}_{NNLO}^S$	$X_3^0 \mathcal{M}_{m+3}^0 ^2$	$X_4^0 \mathcal{M}_{m+2}^0 ^2$	$X_3^0 X_3^0 \mathcal{M}_{m+2}^0 ^2$	$X_3^0 X_3^0 \mathcal{M}_{m+2}^0 ^2$	$SX_3^0 \mathcal{M}_{m+2}^0 ^2$
$\int_1 d\hat{\sigma}_{NNLO}^{S,1}$	$\chi_3^0 \mathcal{M}_{m+3}^0 ^2$	–	$\chi_3^0 X_3^0 \mathcal{M}_{m+2}^0 ^2$	–	$SX_3^0 \mathcal{M}_{m+2}^0 ^2$
$\int_2 d\hat{\sigma}_{NNLO}^{S,2}$	–	$\chi_4^0 \mathcal{M}_{m+2}^0 ^2$	–	$\chi_3^0 \chi_3^0 \mathcal{M}_{m+2}^0 ^2$	–

$$\text{Poles}(d\hat{\sigma}_{NNLO}^{U,a}) \sim \mathbf{J}^1(\epsilon, \hat{1}_g, \hat{2}_g, i_g, j_g) \left(A_4^1(\hat{1}_g, \hat{2}_g, i_g, j_g) - \frac{b_0}{\epsilon} A_4^0(\hat{1}_g, \hat{2}_g, i_g, j_g) \right)$$

$$\text{Poles}(d\hat{\sigma}_{NNLO}^{U,b}) \sim \mathbf{J}^1(\epsilon, \hat{1}_g, \hat{2}_g, i_g, j_g) \otimes \mathbf{J}^1(\epsilon, \hat{1}_g, \hat{2}_g, i_g, j_g) A_4^0(\hat{1}_g, \hat{2}_g, i_g, j_g)$$

$$\text{Poles}(d\hat{\sigma}_{NNLO}^{U,c}) \sim \mathbf{J}^2(\epsilon, \hat{1}_g, \hat{2}_g, i_g, j_g) A_4^0(\hat{1}_g, \hat{2}_g, i_g, j_g)$$

	$d\hat{\sigma}_{NNLO}^T$		
Final State Particles	<i>a</i>	<i>a</i>	(<i>b, c</i>)
$m + 1$	$X_3^1 \mathcal{M}_{m+2}^0 ^2$	$X_3^0 \mathcal{M}_{m+2}^1 ^2$	$\chi_3^0 X_3^0 \mathcal{M}_{m+2}^0 ^2$
m	$\chi_3^1 \mathcal{M}_{m+2}^0 ^2$	$\chi_3^0 \mathcal{M}_{m+2}^1 ^2$	$\chi_3^0 \chi_3^0 \mathcal{M}_{m+2}^0 ^2$

- integrated operators $\mathbf{J}_2^{(2,1)}$ in **analytic** one-to-one correspondence with $(\mathbf{I}_1)^2$ operator of Catani

$$\begin{aligned}
d\sigma_{VV} &= 2\mathbf{I}^{(1)}(\epsilon, \hat{1}_g, \hat{2}_g, i_g, j_g) A_4^1(\hat{1}_g, \hat{2}_g, i_g, j_g) \\
&- 2\mathbf{I}^{(1)}(\epsilon, \hat{1}_g, \hat{2}_g, i_g, j_g)^2 A_4^0(\hat{1}_g, \hat{2}_g, i_g, j_g) \\
&+ \mathbf{I}^{(2)}(\epsilon, \hat{1}_g, \hat{2}_g, i_g, j_g) \\
&+ \text{Finite}(A_2^2(\hat{1}_g, \hat{2}_g, i_g, j_g))
\end{aligned}$$

$$\begin{aligned}
d\sigma_U &= \mathbf{J}_4^1(1_g, 2_g, i_g, j_g) A_4^1(1_g, 2_g, i_g, j_g) \\
&+ \frac{1}{2} \mathbf{J}_4^1(1_g, 2_g, i_g, j_g) \otimes \mathbf{J}_4^1(1_g, 2_g, i_g, j_g) A_4^0(1_g, 2_g, i_g, j_g) \\
&+ \mathbf{J}_4^2(1_g, 2_g, i_g, j_g) A_4^0(1_g, 2_g, i_g, j_g)
\end{aligned}$$

[Joaos-MacBook-Pro:jet pires\$ form autoA4g2XU.frm

FORM 4.1 (Oct 25 2013) 64-bits

Run: Sun Apr 9 17:19:06 2017

#-

poles = 0;

32.20 sec out of 32.54 sec

$$\boxed{\text{Poles} \left(d\hat{\sigma}_{NNLO}^{VV} - d\hat{\sigma}_{NNLO}^U \right) = 0}$$

Pros and cons

Antenna subtraction

- **local method** with **phase space averaging** → good control on the **numerical accuracy** of the final result, RR, RV, VV separately finite
- **analytic** IR **pole cancellation** at NNLO → good control on the **correctness** of the **pole cancellation**
- double precision
- **universal** method works for **general jet multiplicity** → no additional building blocks needed
- $pp \rightarrow jj, H_j, Z_j$ @ NNLO
- **subtraction terms** for a **fixed** colour structure **reusable**
- involves many **mappings/subtraction terms** as expected for a **local method** → needs caching system to store mappings

NNLOJET parton level generator

[X. Chen, J. Cruz-Martinez, J. Currie, A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, A. Huss, M. Jaquier, T. Morgan, J. Niehues, JP]

- parton level generator based on antenna subtraction to compute fully differential cross sections at NNLO in QCD
- all antenna functions implemented with a common syntax and structure
- allows testing of matrix element/subtraction term cancellations with singular phase space trajectories
- allows explicit $1/\epsilon$ pole cancellation with one and two loop matrix elements with FORM
- interface to phase space generation, Monte Carlo integration and fully flexible histogramming
- interface to applfast nnlo tables in development

M. Sutton and K. Rabbertz tomorrow afternoon

NNLOJET parton level generator

[X. Chen, J. Cruz-Martinez, J. Currie, A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, A. Huss, M. Jaquier, T. Morgan, J. Niehues, JP]

- list of processes available in NNLOJET
 - $pp \rightarrow H, W, Z$
 - $pp \rightarrow H + \text{jet}$
 - $pp \rightarrow Z + \text{jet}$
 - $pp \rightarrow 2 \text{ jets}$
 - $ep \rightarrow 2 \text{ jets}$
 - ...

Single jet inclusive cross section

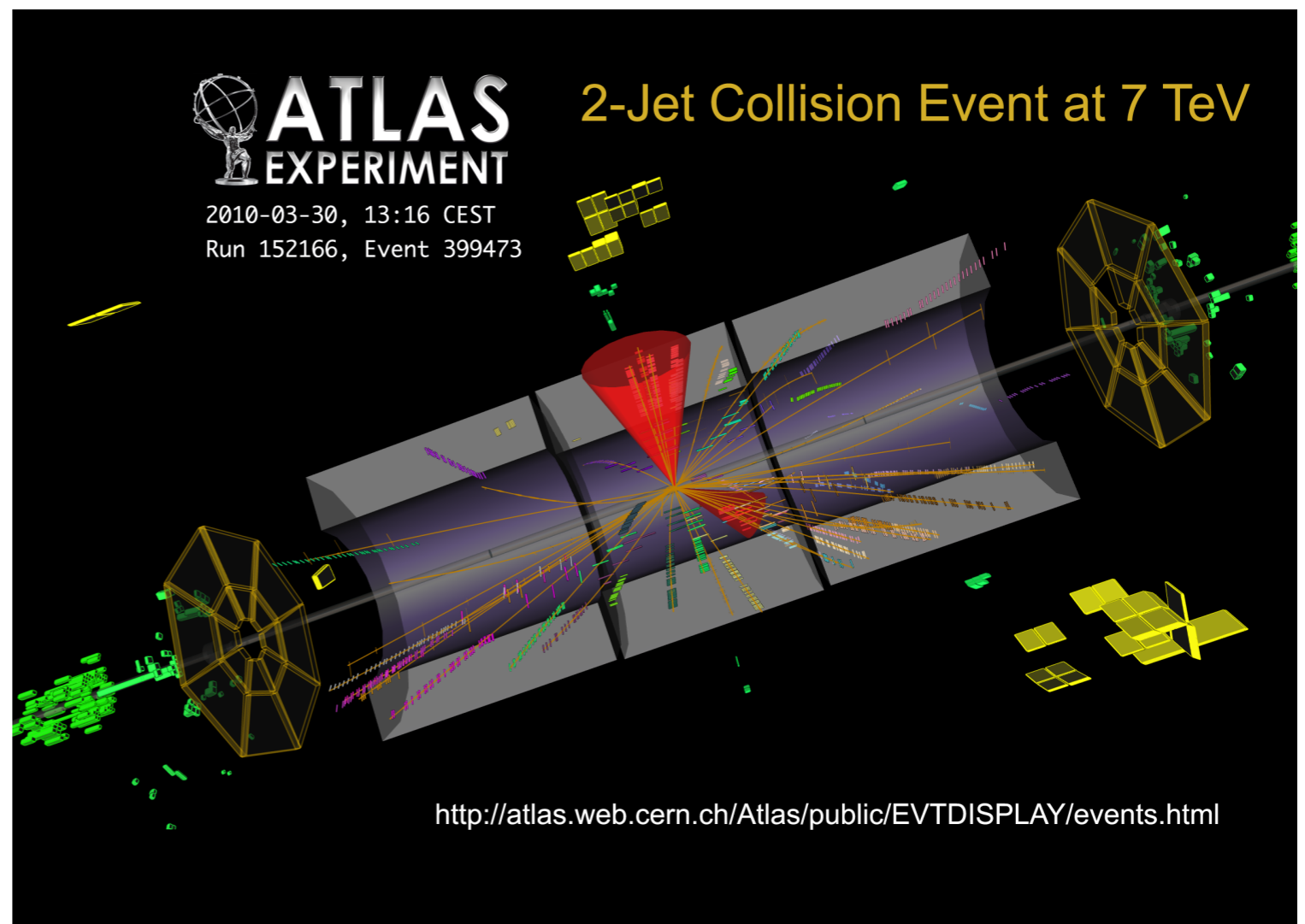
ATLAS jets

Theory setup

- NNPDF3.0_nnlo
- anti- k_T jet algorithm
- $\mu_R = \mu_F = \{p_{T1}, p_T\}$
- vary scales by factors of 2 and 1/2

Comparison to data

- ATLAS 7 TeV 4.5 fb⁻¹
- R=0.4

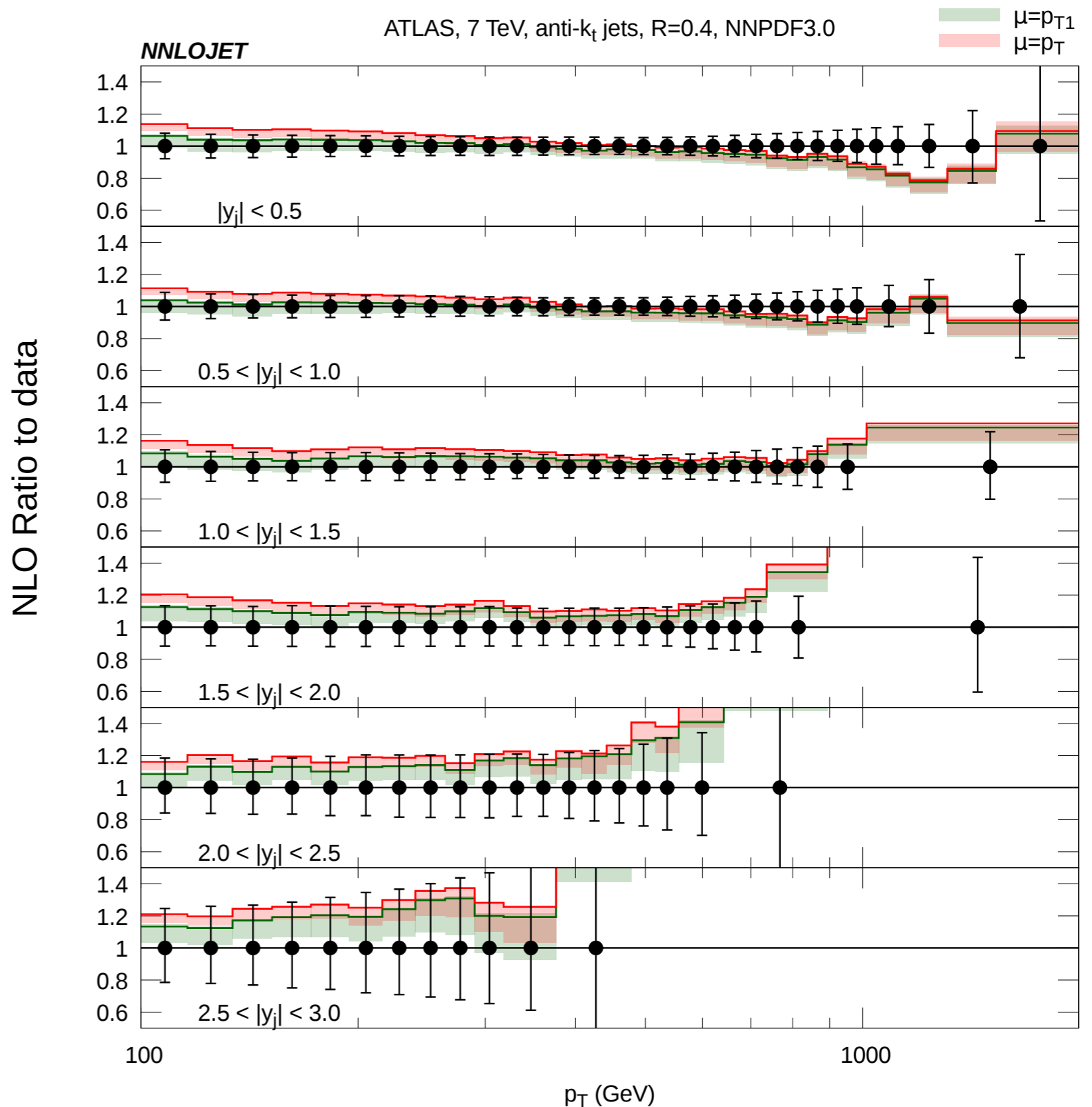


Ratio to NLO

- asymmetric scale band variation
- underestimated at small p_T due to turn over of the NLO coefficient
- 20% uncertainty for central high p_T jets rising to 40% for forward jets

Comparison to data

- non perturbative effects $< 2\%$ effect [JHEP 1509, 141 (2015)]
- data favours the p_{T1} scale choice at NLO

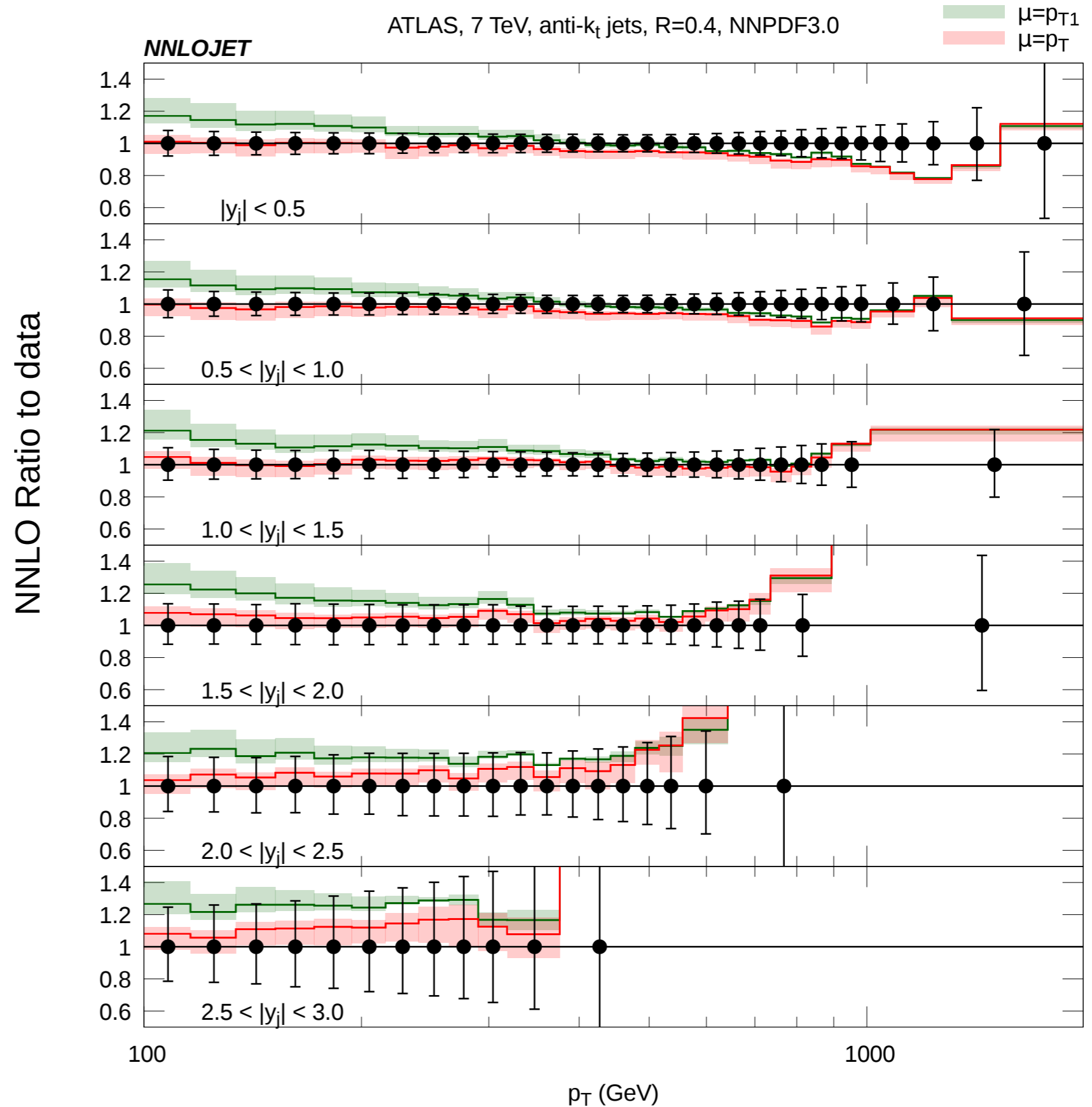


Ratio to NNLO

- symmetric scale band variation
- $p_{T1} \neq p_T$ effects enlarged at NNLO
- 10% scale uncertainty at low p_T and percent level scale uncertainty at high p_T

Comparison to data

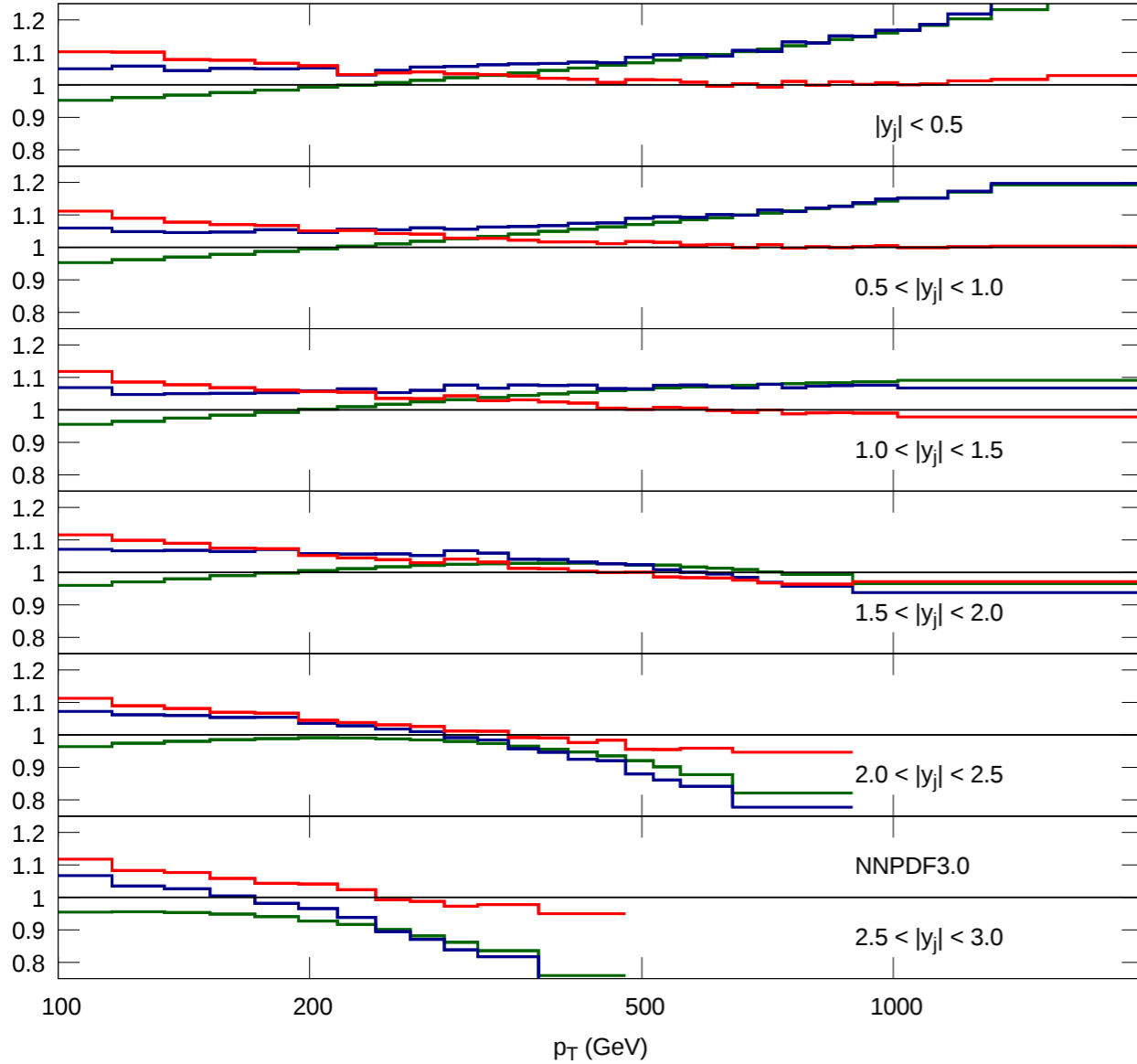
- data favours the p_T scale choice at NNLO



ATLAS, 7 TeV, anti- k_t jets, R=0.4

— NLO/LO
— NNLO/LO
— NNLO/NLO

NNLOJET

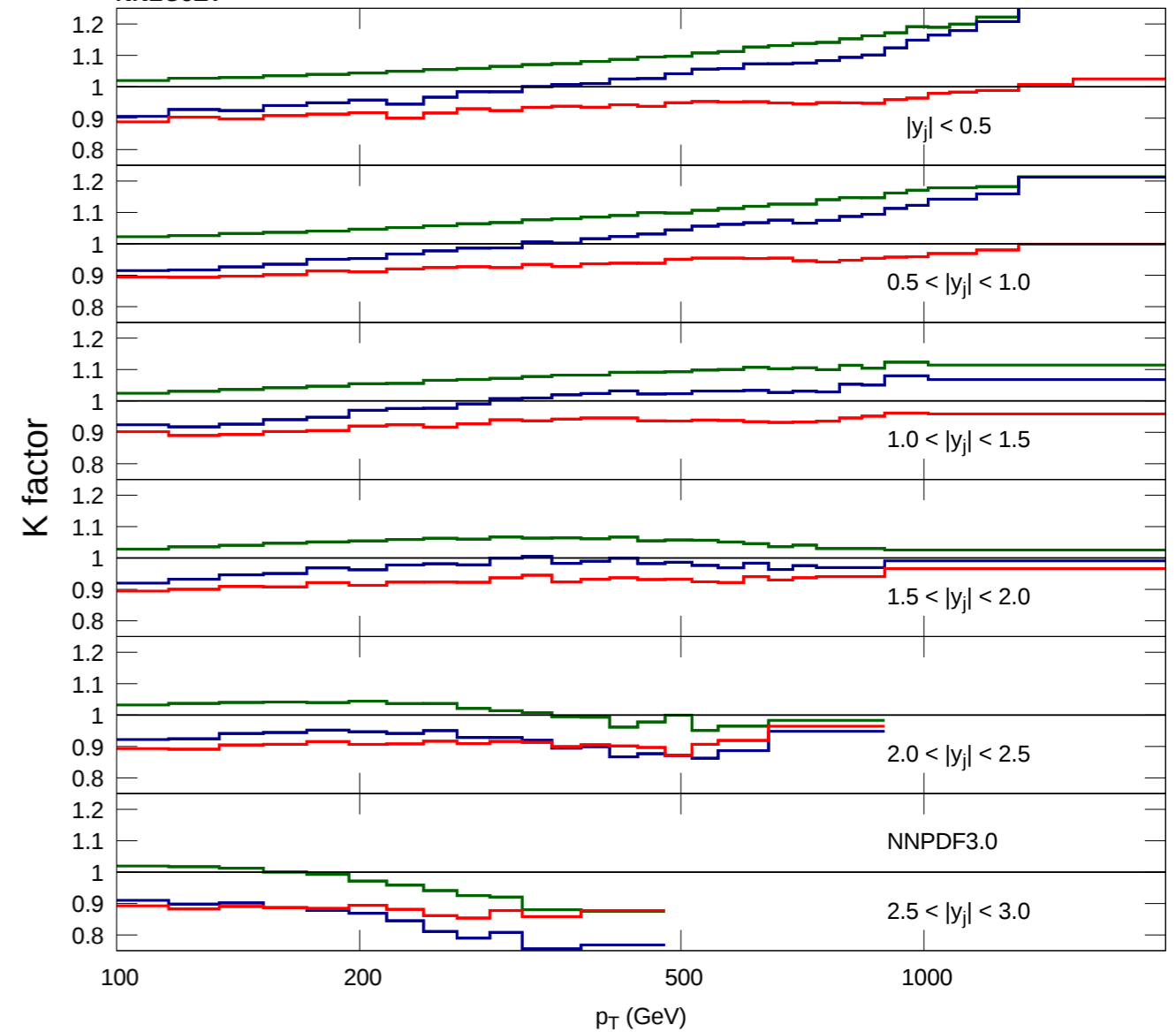


$$\mu_R = \mu_F = p_{T1}$$

ATLAS, 7 TeV, anti- k_t jets, R=0.4

— NLO.pt/LO
— NNLO.pt/LO
— NNLO.pt/NLO

NNLOJET

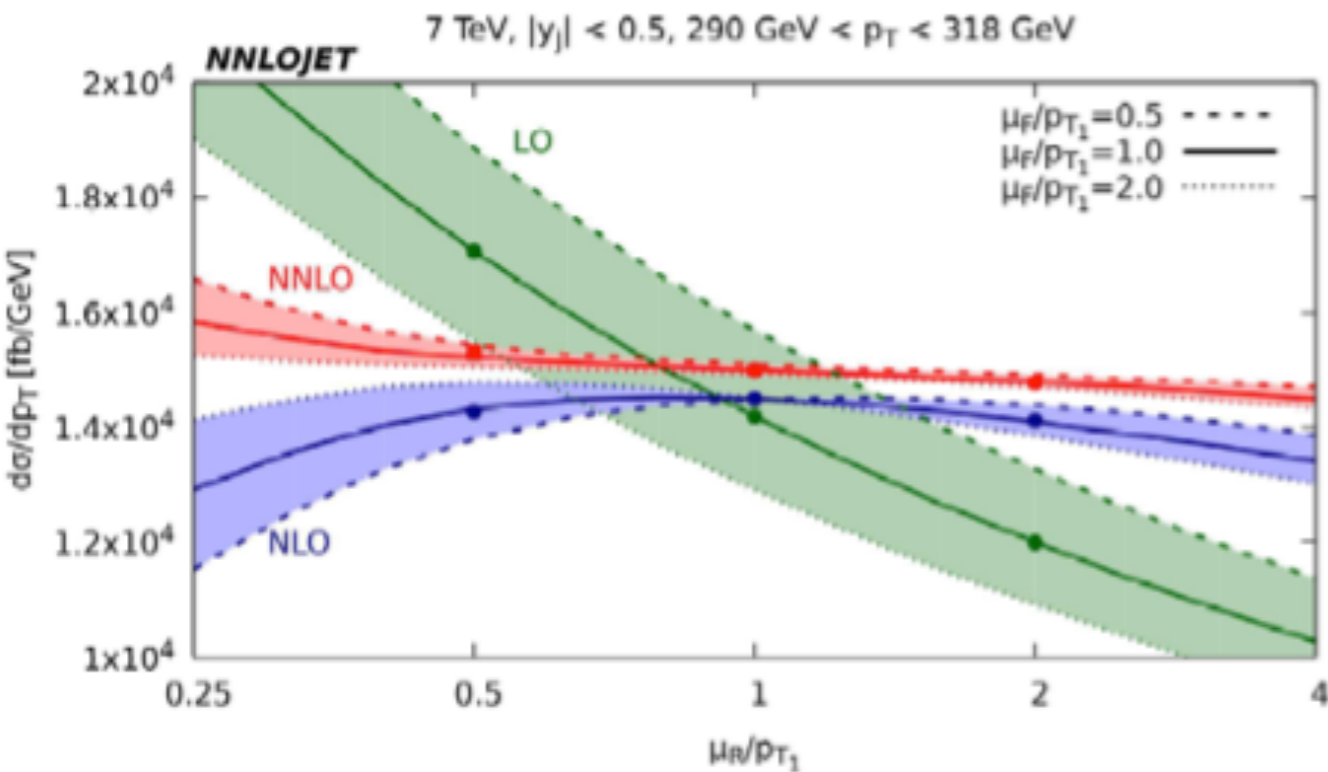


$$\mu_R = \mu_F = p_T$$

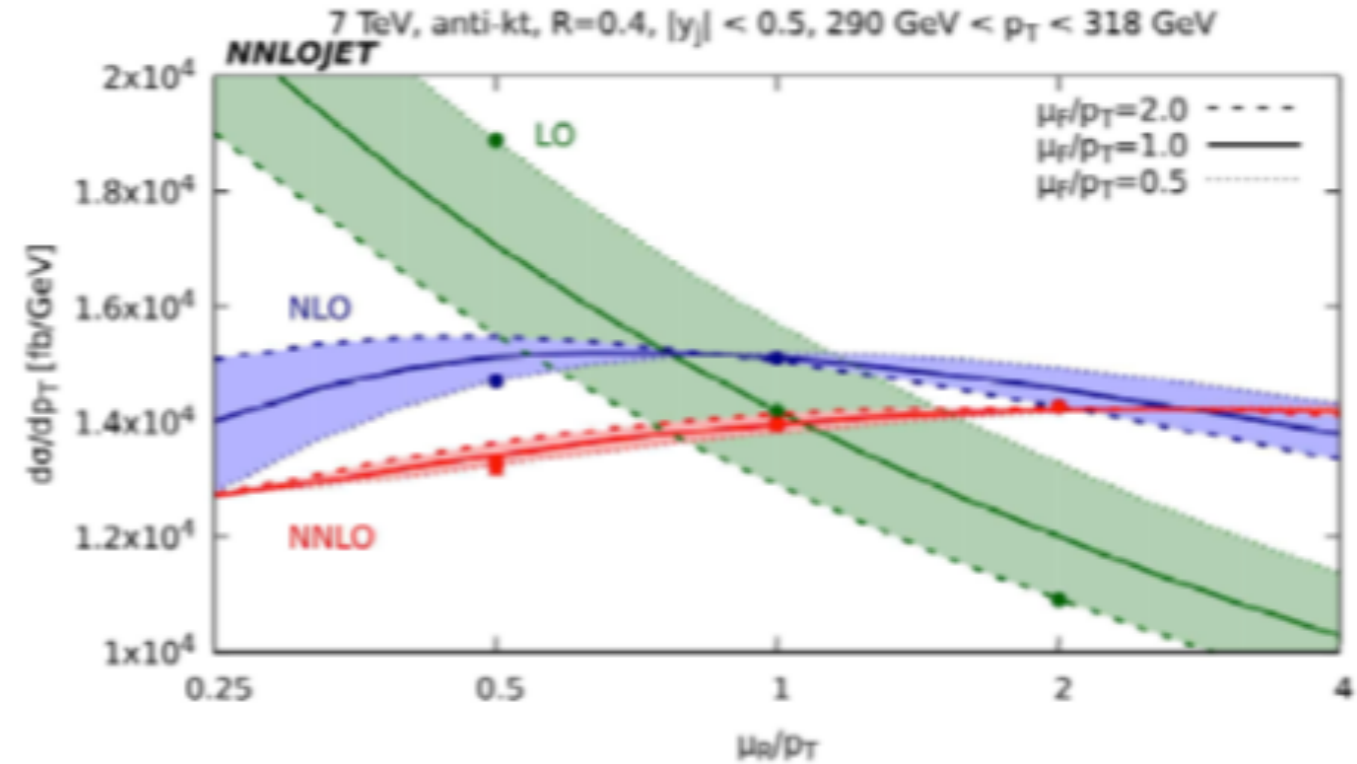
- NNLO effects around +10% at low p_T and small at high p_T
- Shape of NNLO/NLO k-factor is getting steeper going to the forward rapidity slices
 - Scale choice has a potential interplay with consistent fit of jet data in PDF's for all rapidity slices

- NNLO effects around -10% at low p_T and small at high p_T
- Shape of NNLO/NLO k-factor is getting flatter going to the forward rapidity slices

Scale variation



$$\mu_R = \mu_F = p_{T1}$$



$$\mu_R = \mu_F = p_T$$

- Different behaviour in the NNLO scale variation
- Scale uncertainty much smaller than the difference between the two scale choices
- Difference in the prediction with either scale choice is beyond the scale variation uncertainty
- Lack of a theoretically well motivated preference motivates further study of this issue

Dijet inclusive cross section

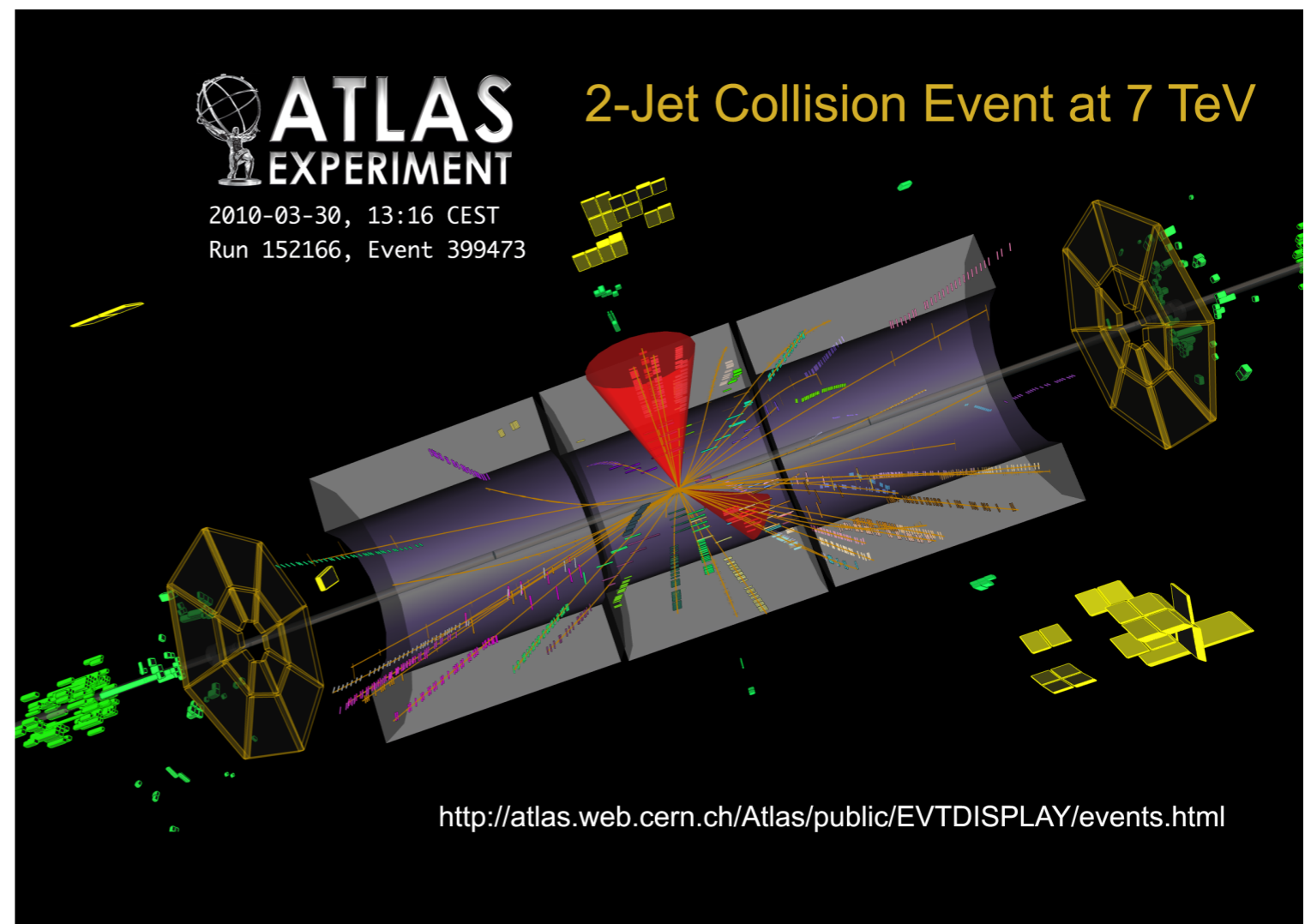
ATLAS jets

Theory setup

- MMHT2014 nnlo
- anti- k_T jet algorithm
- $p_{T1} > 100$ GeV; $p_{T2} > 50$ GeV;
- $|y_{j1}|, |y_{j2}| < 3.0$
- $\mu_R = \mu_F = \{m_{jj}, \langle p_T \rangle\}$
- vary scales by factors of 2 and 1/2

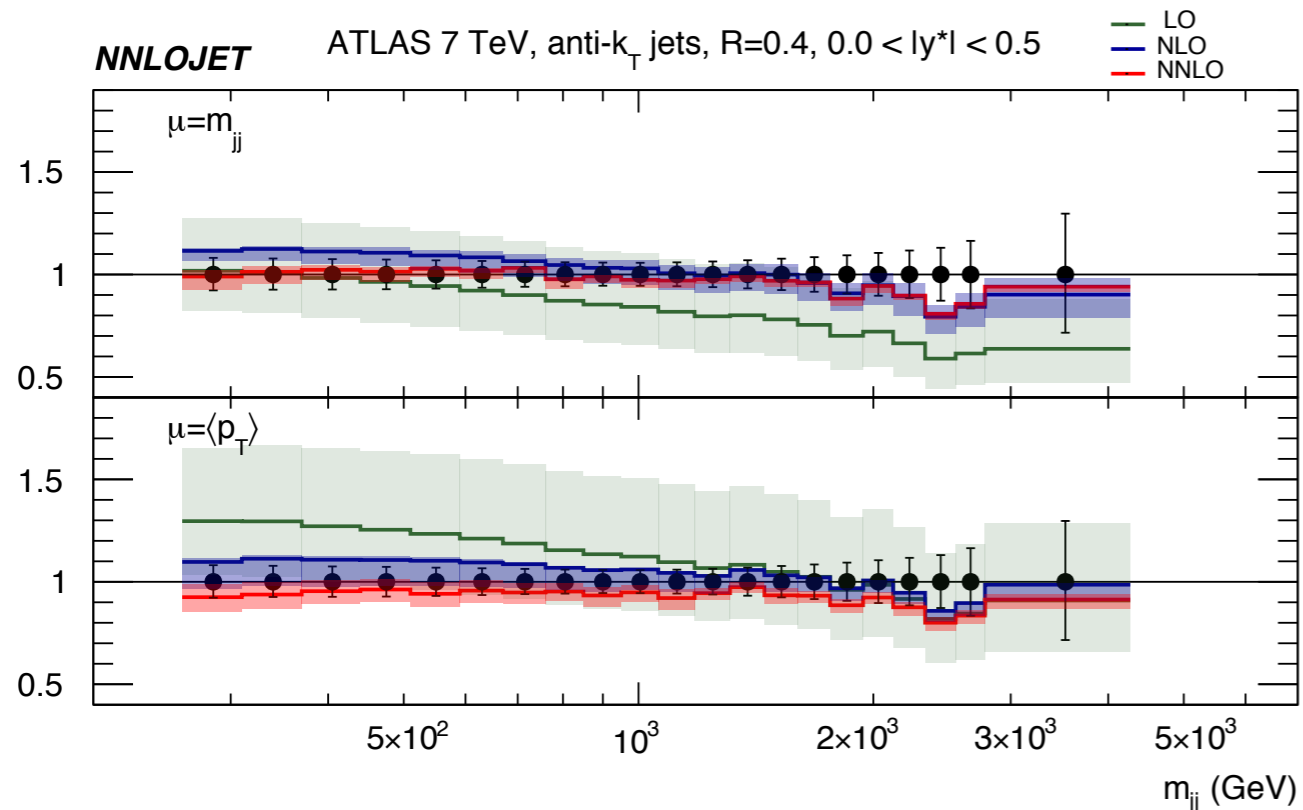
Comparison to data

- ATLAS 7 TeV 4.5 fb^{-1}
- $R=0.4$

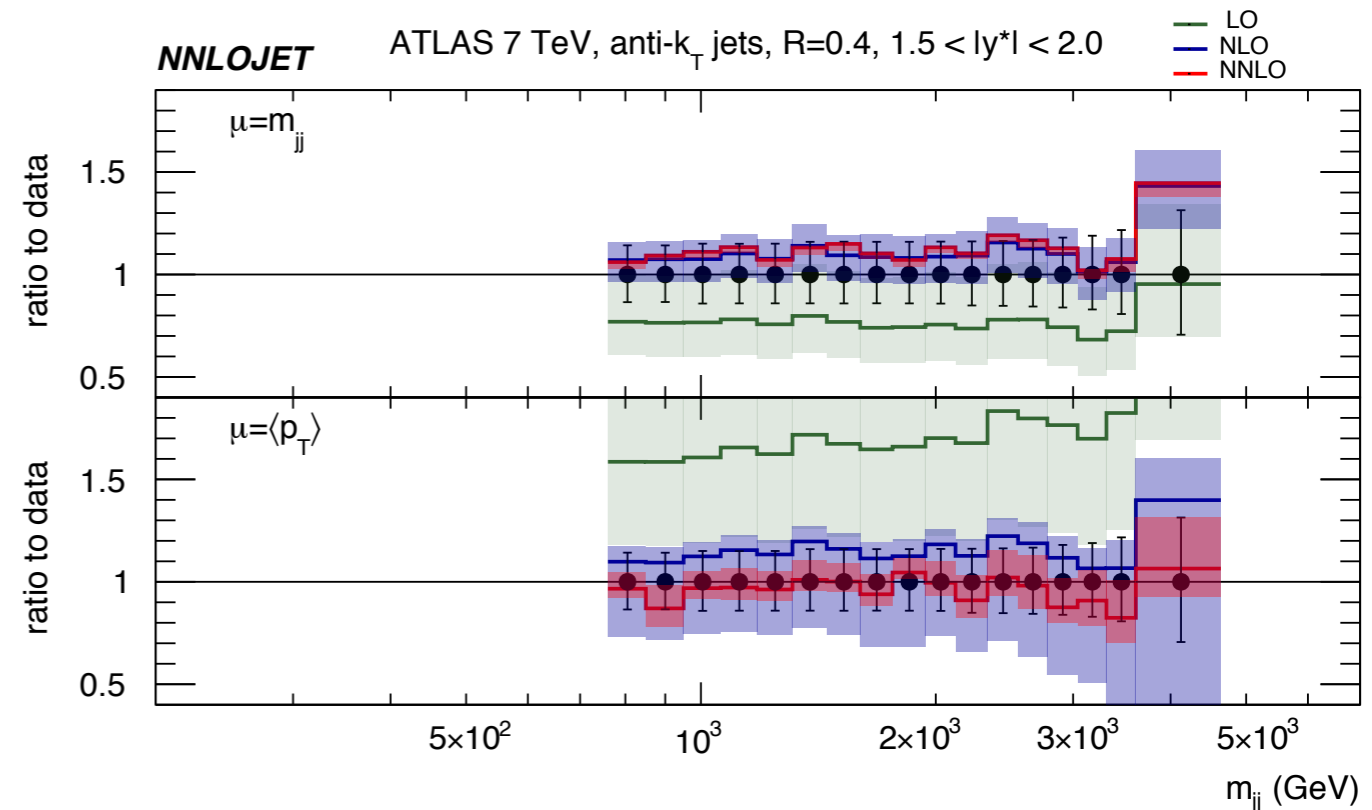


$$m_{jj}^2 = (p_{j1} + p_{j2})^2$$

$$y^* = \frac{1}{2}(y_{j1} - y_{j2})$$

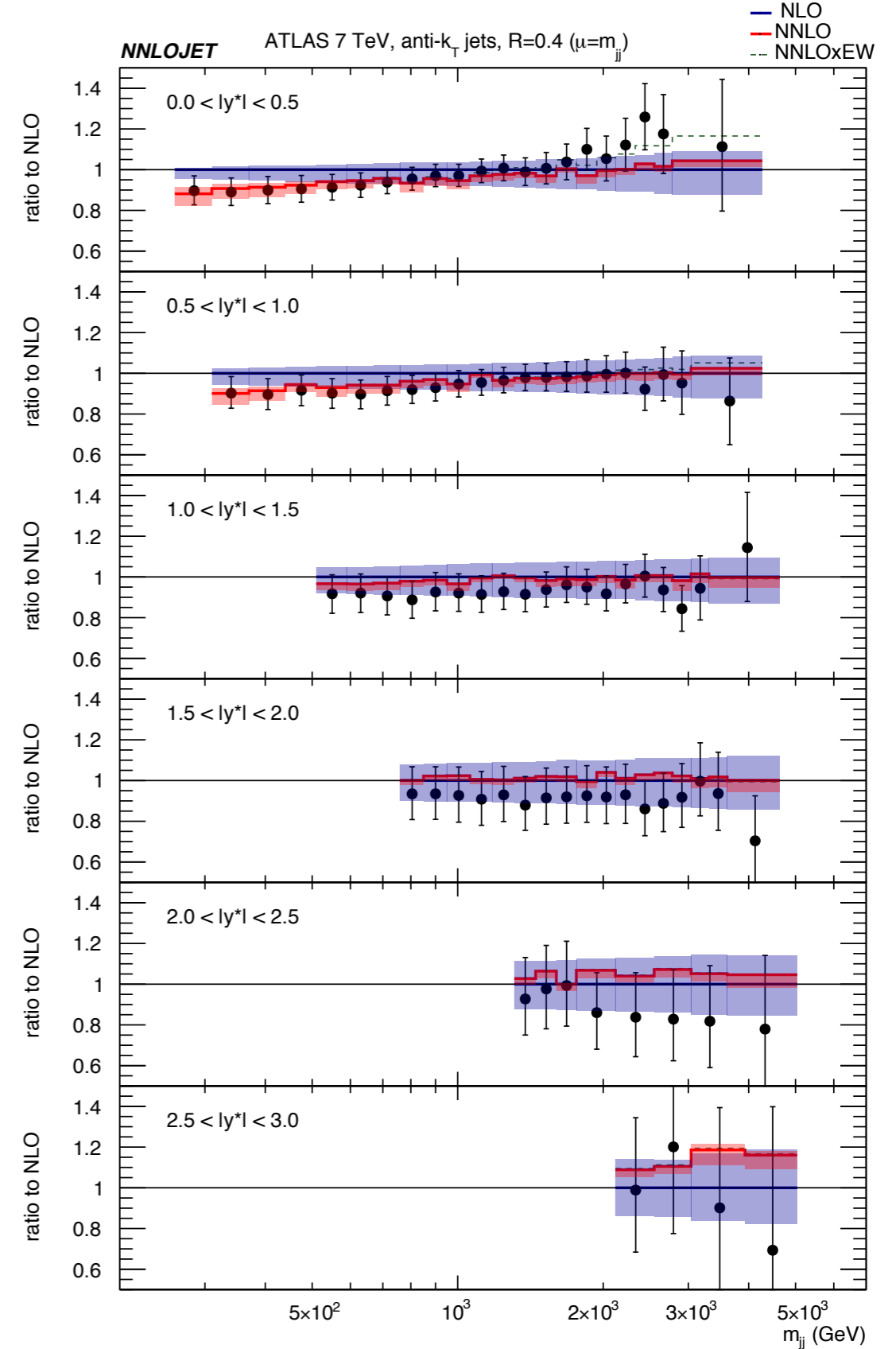
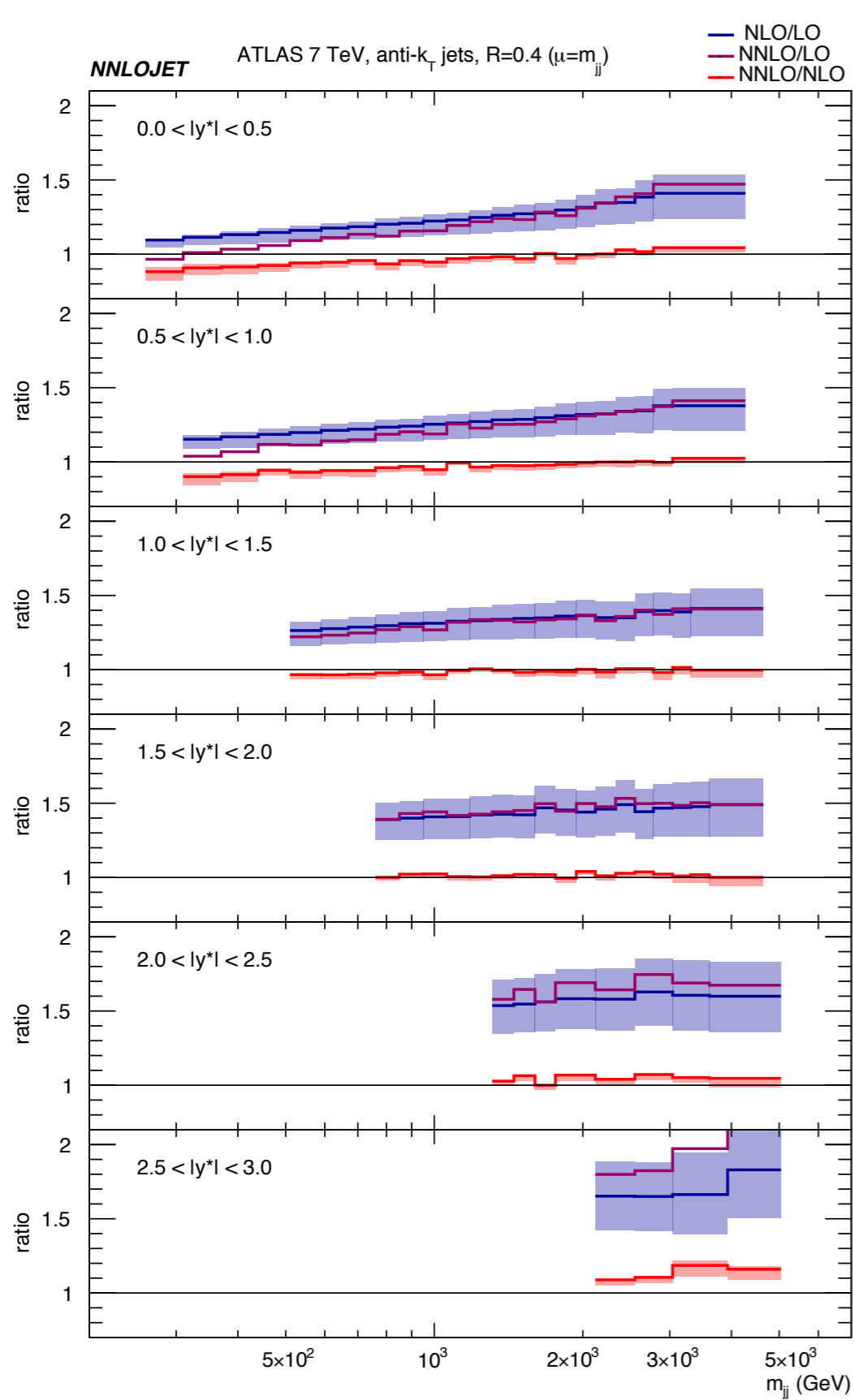


$$0.0 < |y^*| < 0.5$$



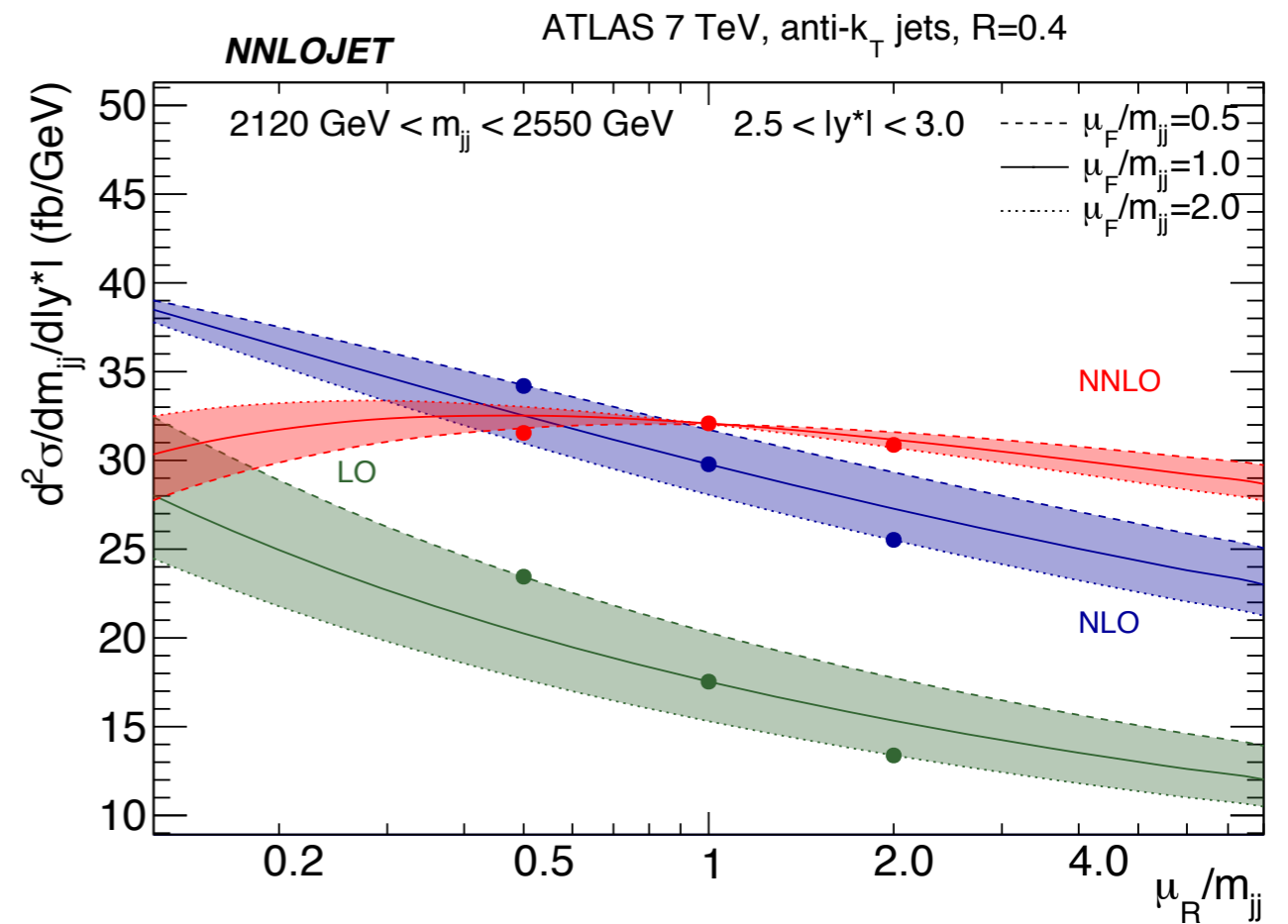
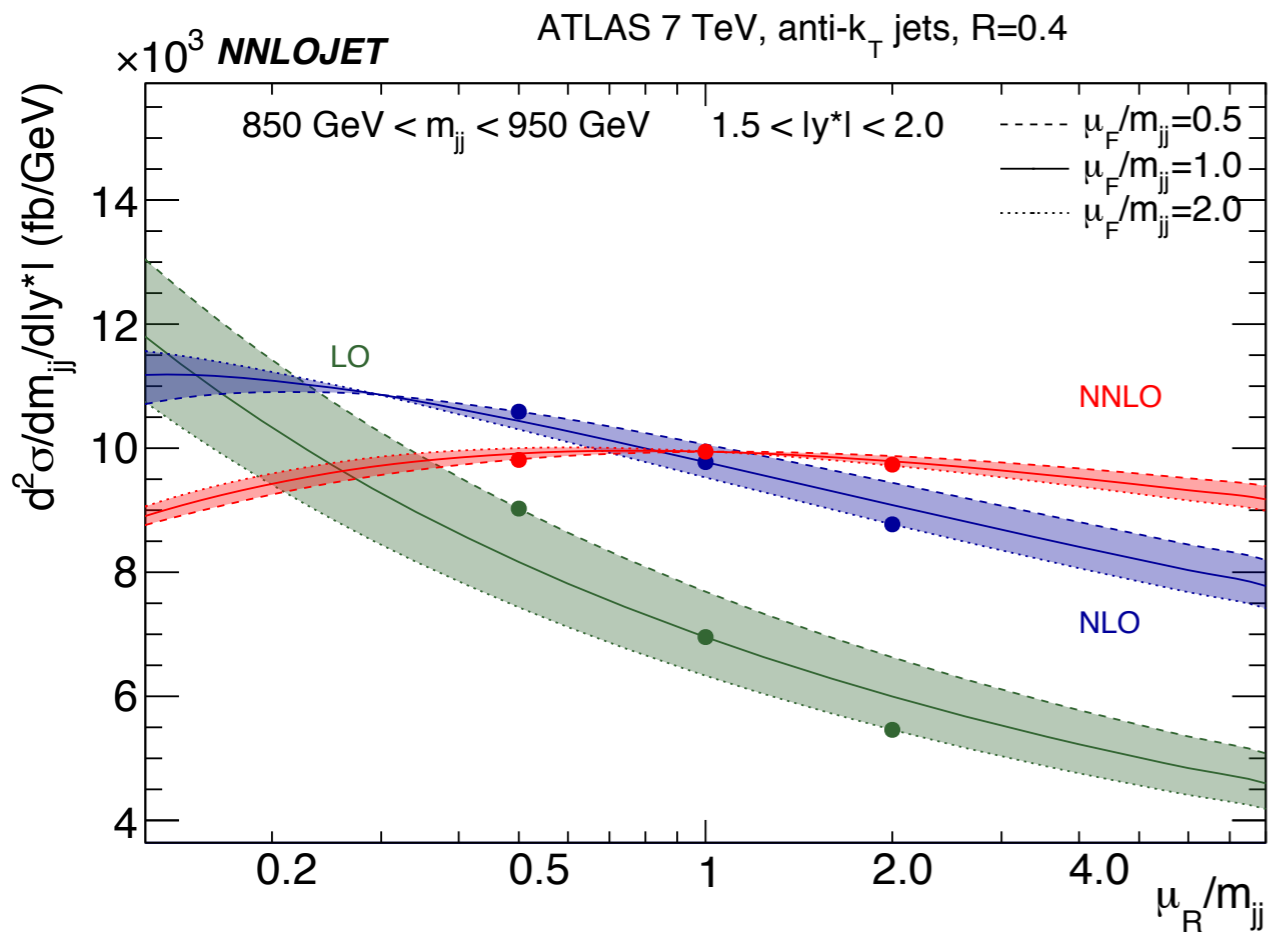
$$1.5 < |y^*| < 2.0$$

- Largely overlapping scale bands at small y^* with either scale choice
- At large y^* we observe with $\mu = \langle P_T \rangle$ large negative NLO corrections, non-overlapping scale bands and residual NLO, NNLO scale uncertainty of $\sim 100\%$, $\sim 20\%$
- Good theoretical motivation to use $\mu = m_{jj}$ as central scale choice



- Excellent convergence of the perturbative expansion; NNLO/NLO $< 10\%$ and flat
- Improved description of the dijet data at NNLO

Scale variation



- Overlapping NLO and NNLO scale bands
- Significant reduction in scale dependence of the prediction at NNLO
- Residual scale uncertainty <5% smaller than experimental uncertainty on the observable

Conclusions

Substantial progress in NNLO calculations in past couple of years

- several different approaches for isolating IR singularities
- several new calculations available

Antenna subtraction

- local IR phase space subtraction scheme with analytic pole cancellation
- RR, RV, VV contributions separately finite and integrable in $d=4$
- formalism implemented in a fully flexible parton level generator
- new processes can be added using the existing common syntax structures
- distribution of results via applfast interface (to APPLGRID and fastNLO)



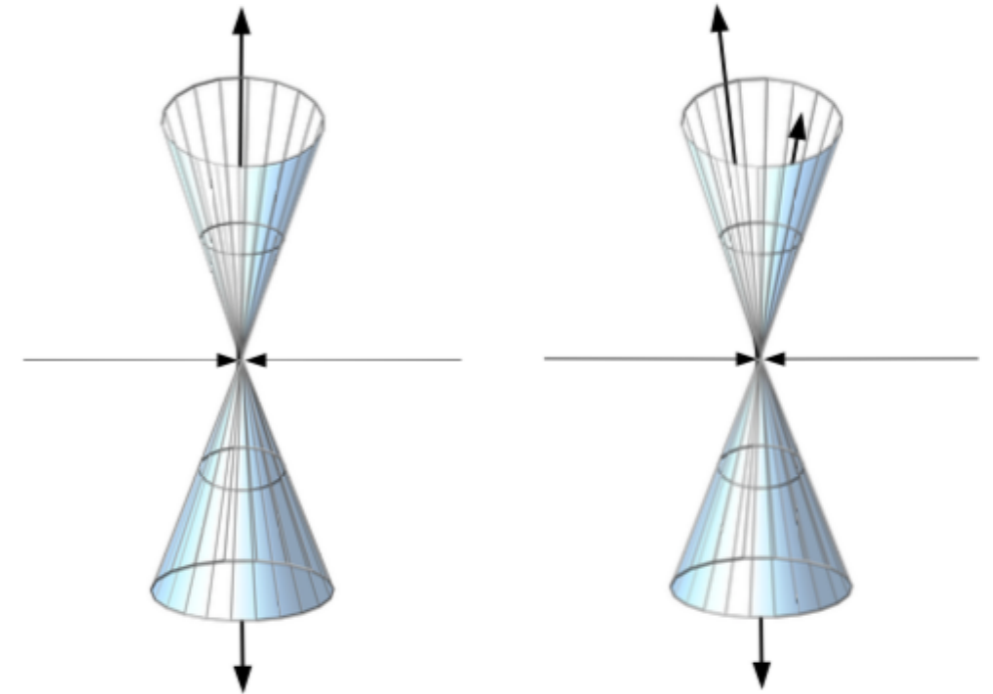
M. Sutton and K. Rabbertz tomorrow afternoon

BACKUP

Single jet inclusive scale choice

two widely used scale choices:

- $\mu_R = \mu_F = \{p_{T1}, p_T\}$
 - leading jet p_T in the event p_{T1}
 - individual jet p_T
- high p_T jets are back to back $\Rightarrow p_T \rightarrow p_{T1}$



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 - leading jet p_T in the event p_{T1}
 - individual jet p_T
 - high p_T jets are back to back $\Rightarrow p_T \rightarrow p_{T1}$
 - $p_T \neq p_{T1}$ for:
 - 3jet events
 - 3rd jet outside fiducial jet cuts
- \Rightarrow with p_T choice the real emission event with different R gives rise to a different scale \Rightarrow larger $R \Rightarrow$ harder scale $\Rightarrow p_T \rightarrow p_{T1}$
- at NLO the p_{T1} scale choice generates the same hard scale for the event independent of the value of R

