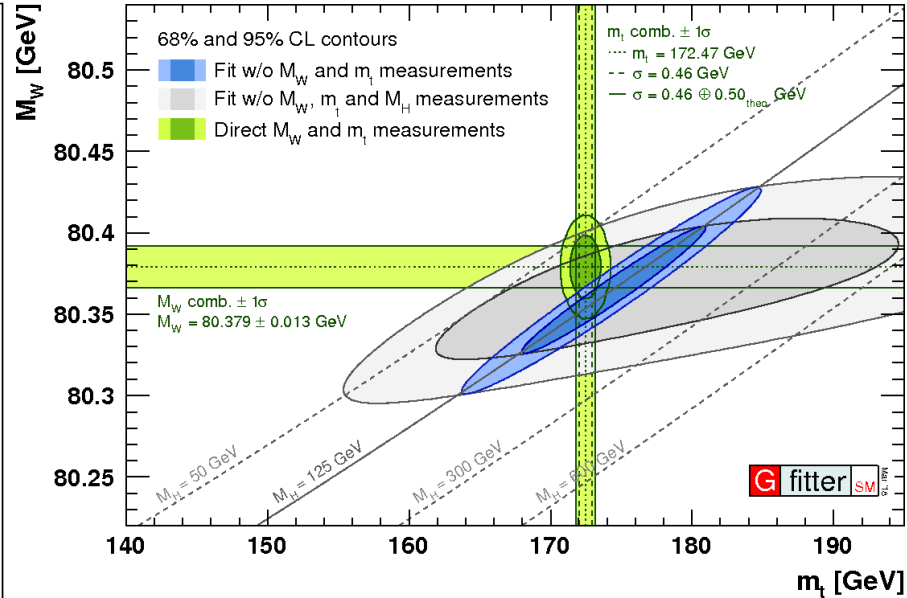
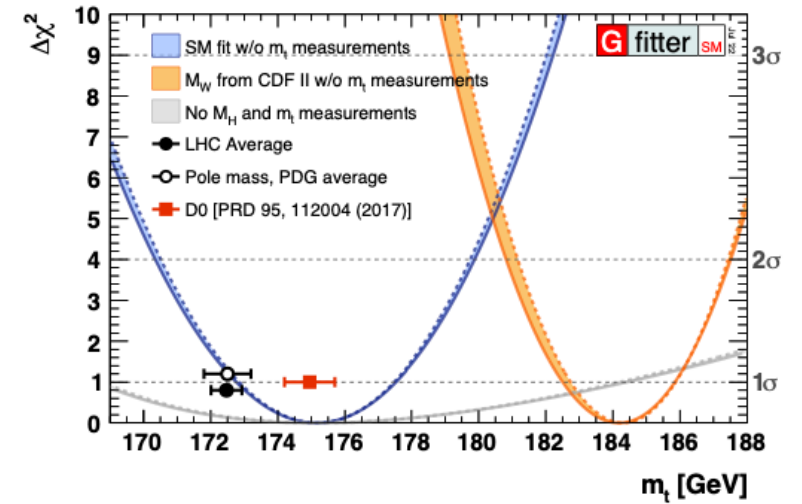
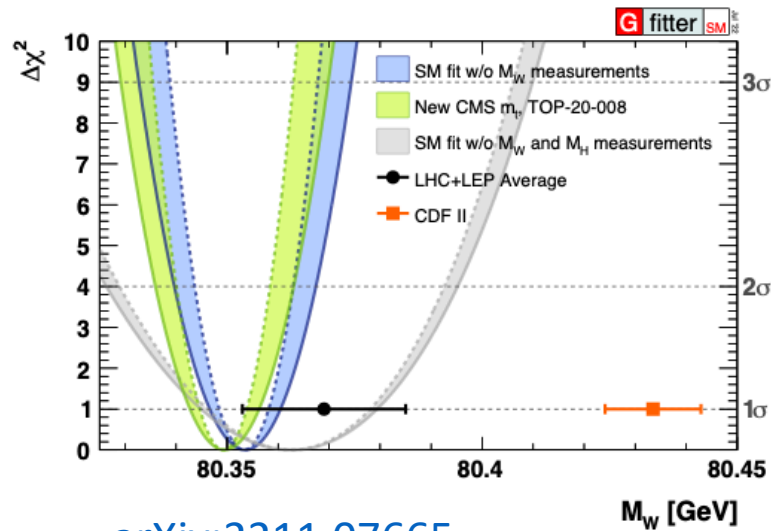
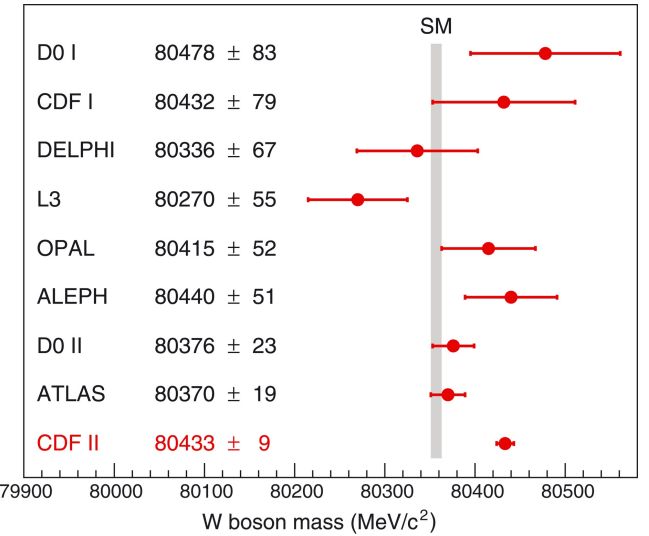


arXiv:1803.01853



Science 376 (2022) 170



arXiv:2211.07665

EW fit $p = 0.34 \rightarrow 10^{-7} (\gtrsim 5\sigma)$

W mass discussion

SM phenomenology group
(J. Huston, A. Huss, M. Pellen, P. Azzurri)

- (non-perturbative) modeling
- new ideas/methods (asymmetry)
- determination at e^+e^-
- theory agnostic determination; how agnostic? (Tanmay Sarkar)

Future e+e- collider measurements of the W mass & width

from pervious presentations and: *The W mass and width
measurement challenge at FCC-ee* [arXiv:2107.04444](https://arxiv.org/abs/2107.04444)

future e+e- mW digest

1. from WW **threshold** cross sections at $E_{\text{CM}} \simeq 157.5\text{-}162.5$ GeV
 $\rightarrow \Delta m_W = 0.3$ MeV [10/ab]

Syst : Theory calculations / E_{CM} / acceptance / background

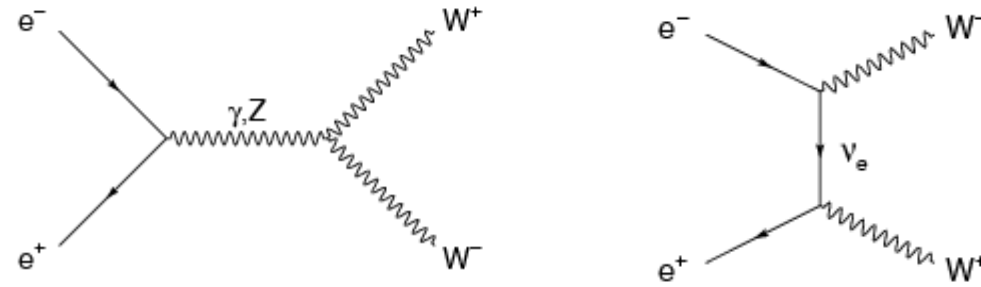
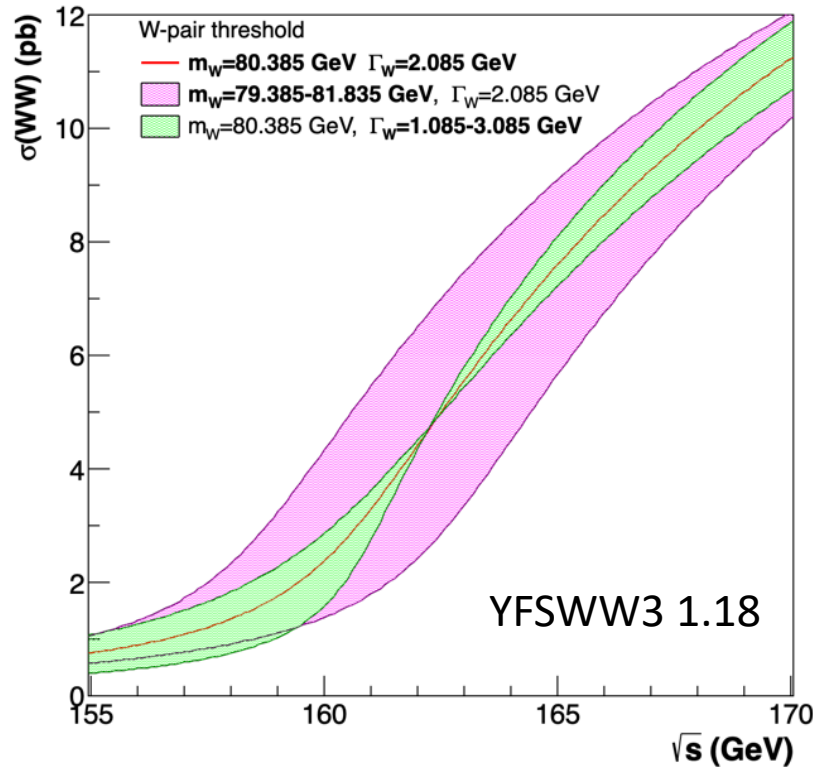
2. from decay **kinematics** mostly at $E_{\text{CM}} \simeq 240$ GeV **and** E_{beam} (LEP2)
 $\rightarrow \Delta m_W = 1\text{-}0.5$ MeV (stat) [2-5/ab] : **2-5 MeV (syst) ?**

Syst : Theory modeling (NP QCD) / E_{CM} / det calibration /

3. from **lepton decay kinematics** and hadronic decays **without** E_{beam}
 $\rightarrow \Delta m_W = 2$ MeV (stat) : **2-5 MeV (tot) ?**

Syst : det calibration / Theory modeling (NP QCD)

The WW threshold lineshape and the W mass



WW cross section rise $\beta = \sqrt{1 - 4m_W^2/s}$ driven by t-channel production

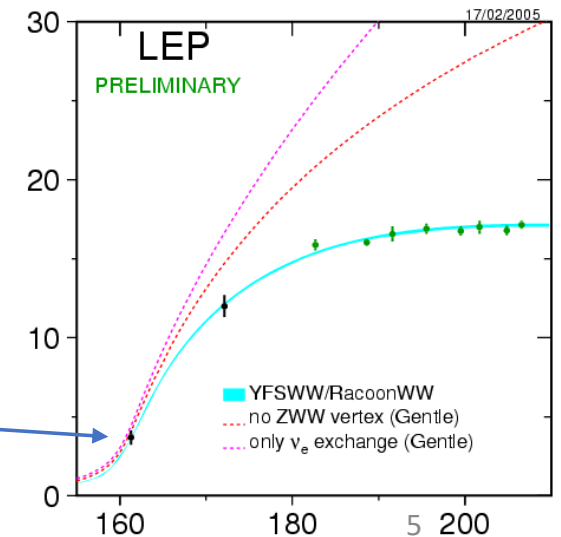
Extract the W mass inverting the m_W dependence

$$\sigma(m_W, E)$$

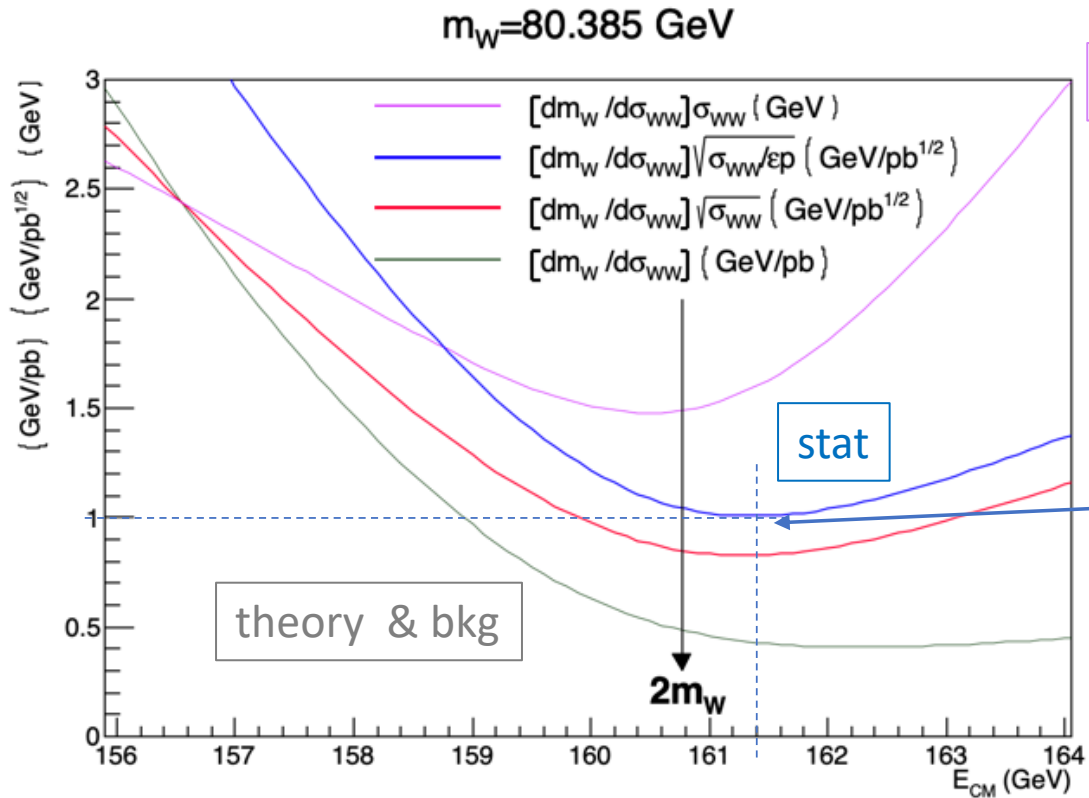
$$m_W = \sigma^{-1}(E)$$

$$\Delta m_W = \left(\frac{d\sigma}{dm_W} \right)^{-1} \Delta\sigma$$

[ALEPH Phys.Lett.B 401 \(1997\) 347](#) with 10/pb $m_W = 80.14 \pm 0.34$ GeV
 stat extrapolation to 10/ab $\Rightarrow \Delta m_W = 0.34$ MeV



The WW threshold : W mass optimal E_{CM}



optimal for stat is also close to optimal for syst contributions

acceptance & lumi

stat uncertainty assuming event selection quality
 $Q = \sqrt{\epsilon p}$ with fixed $\epsilon = 0.75$ and $\sigma_B = 0.3 \text{ pb}$

Max stat sensitivity at $E_{CM} \sim 2m_W + 0.6 \text{ GeV}$

$$\left[\left(\frac{d\sigma}{dm_W} \right)^{-1} \frac{\sqrt{\sigma}}{\sqrt{\epsilon p}} \right]_{min} \cong 1 \frac{\text{GeV}}{\text{pb}^{1/2}} = 1 \frac{\text{MeV}}{\text{ab}^{1/2}}$$

With $L = 12/\text{ab} \Rightarrow \Delta m_W(\text{stat}) = 0.3 \text{ MeV}$

The WW threshold : W mass uncertainties

$$\sigma = \left(\frac{N}{L} - \sigma_B \right) \frac{1}{\varepsilon}$$

$$\Delta m_W(\text{stat}) = \left(\frac{d\sigma}{dm_W} \right)^{-1} \frac{\sqrt{\sigma}}{\sqrt{L}} \frac{1}{\sqrt{\varepsilon p}}$$

Statistical

$$\Delta \sigma_{WW} = \frac{\Delta \sigma_B}{\varepsilon}$$

$$\Delta m_W(B) = \left(\frac{d\sigma}{dm_W} \right)^{-1} \left(\frac{\Delta \sigma_B}{\varepsilon} \oplus \Delta \sigma_{TH} \right)$$

Background and Theory

$$\Delta \sigma_{WW} = \sigma \left(\frac{\Delta \varepsilon}{\varepsilon} \oplus \frac{\Delta L}{L} \right)$$

$$\Delta m_W(\varepsilon) = \left(\frac{d\sigma}{dm_W} \right)^{-1} \left(\frac{\Delta \varepsilon}{\varepsilon} + \frac{\Delta L}{L} \right)$$

Acceptance and Luminosity

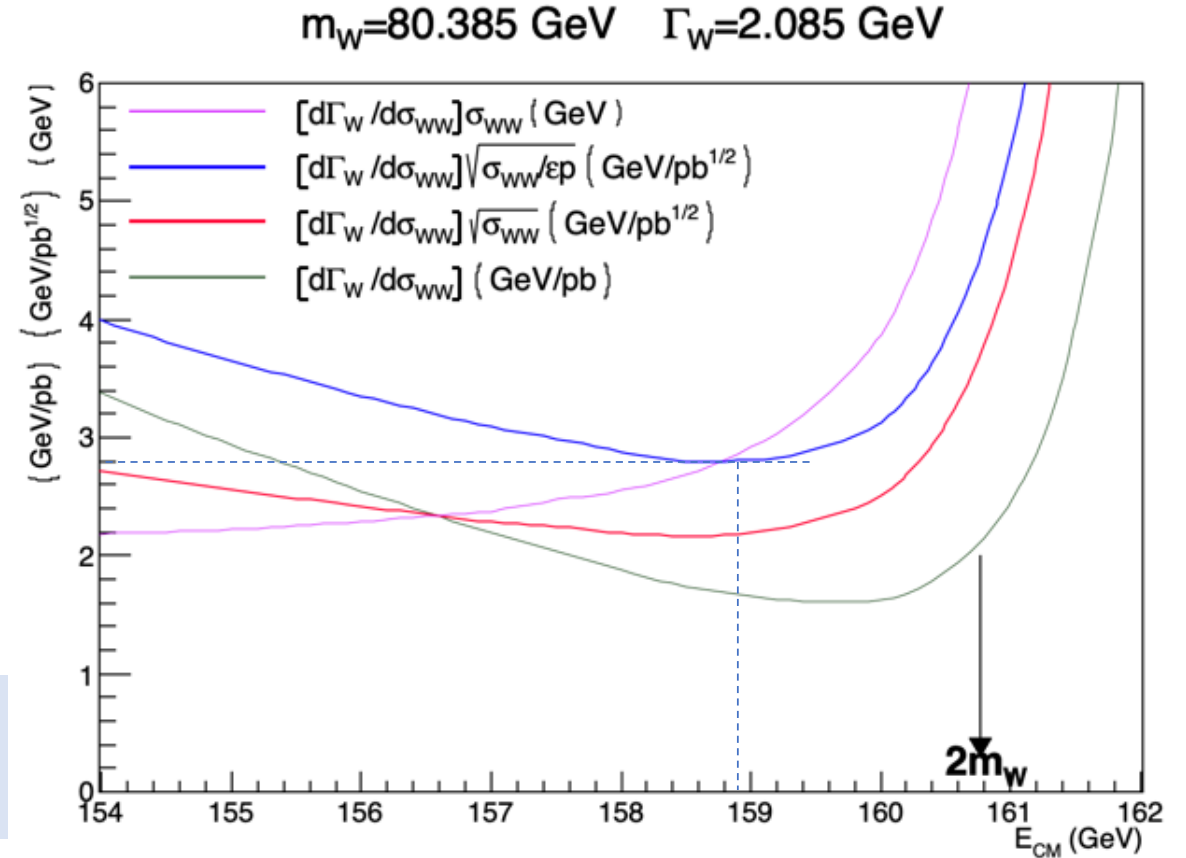
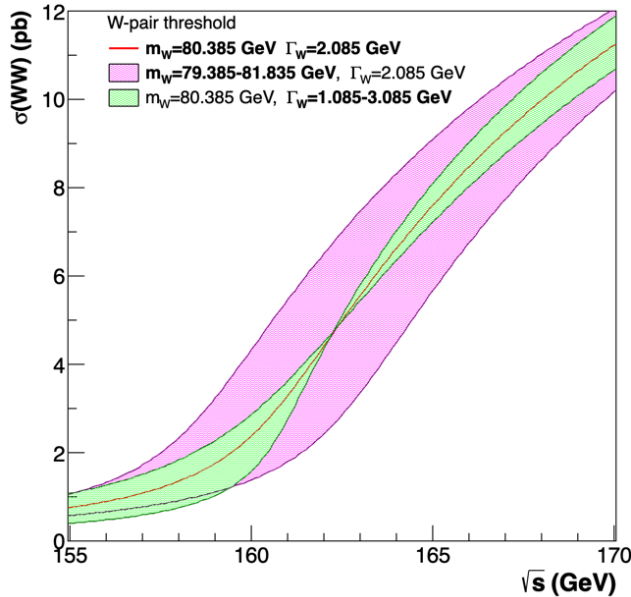
$$\Delta m_W(E) = \left(\frac{d\sigma}{dm_W} \right)^{-1} \left(\frac{d\sigma}{dE} \right) \Delta E \leq \frac{1}{2} \Delta E$$

Collision energy

WW threshold : W mass and width

[arXiv:1703.01626](https://arxiv.org/abs/1703.01626)

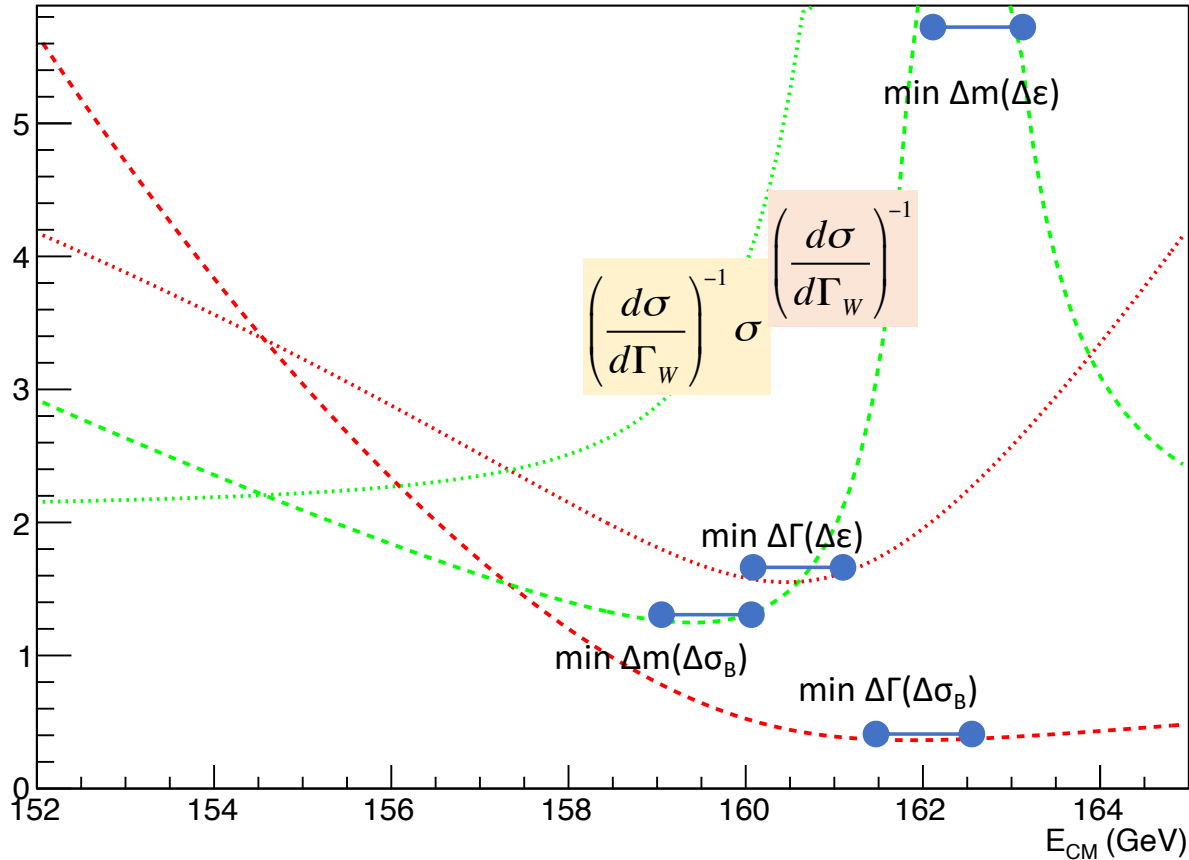
[arXiv:2107.04444](https://arxiv.org/abs/2107.04444)



Max stat sensitivity at $E_{CM} \sim 2m_W - \Gamma_W$

$$\left[\left(\frac{d\sigma}{d\Gamma_W} \right)^{-1} \frac{\sqrt{\sigma}}{\sqrt{\epsilon p}} \right]_{min} \cong 2.8 \frac{GeV}{pb^{1/2}} = 2.8 \frac{MeV}{ab^{1/2}}$$

WW threshold : W mass and width



Scans of (E_1, E_2, f) data taking **assuming limiting syst uncertainties**, either $\Delta\varepsilon + \Delta L$ or $\Delta\sigma_B + \Delta\sigma_{TH}$

More complex situation, depends very much on the correlation of uncertainties between the energy points (that can be quite large)

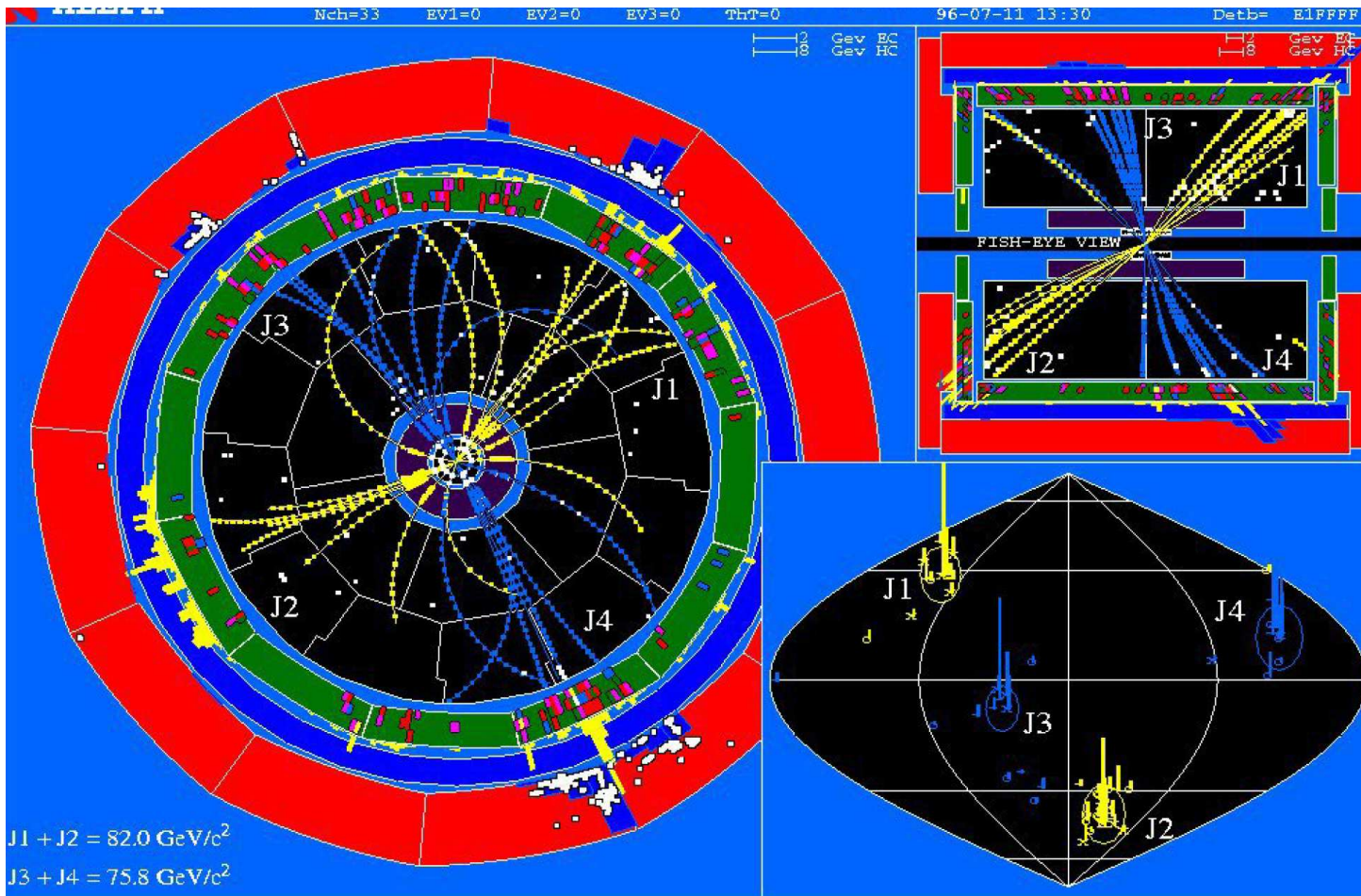
Correlated syst can cancel taking data at different E_{CM} points where the relevant differential factors are equal (around their minima)

>2 energy points will be beneficial to reduce the impact of (correlated) systematic uncertainties
careful choice of additional points recommended

optimal E points with limiting correlated systs

partially explored in [Eur. Phys. J. C 80 no. 1, \(2020\) 66](https://doi.org/10.1007/s00527-019-0460-1)

W mass from decay kinematics



Threshold four jet event

W mass from kinematics with 4P fit (LEP2)

Formula for 2-jets final state from $ee \rightarrow Z\gamma \rightarrow qq\gamma$

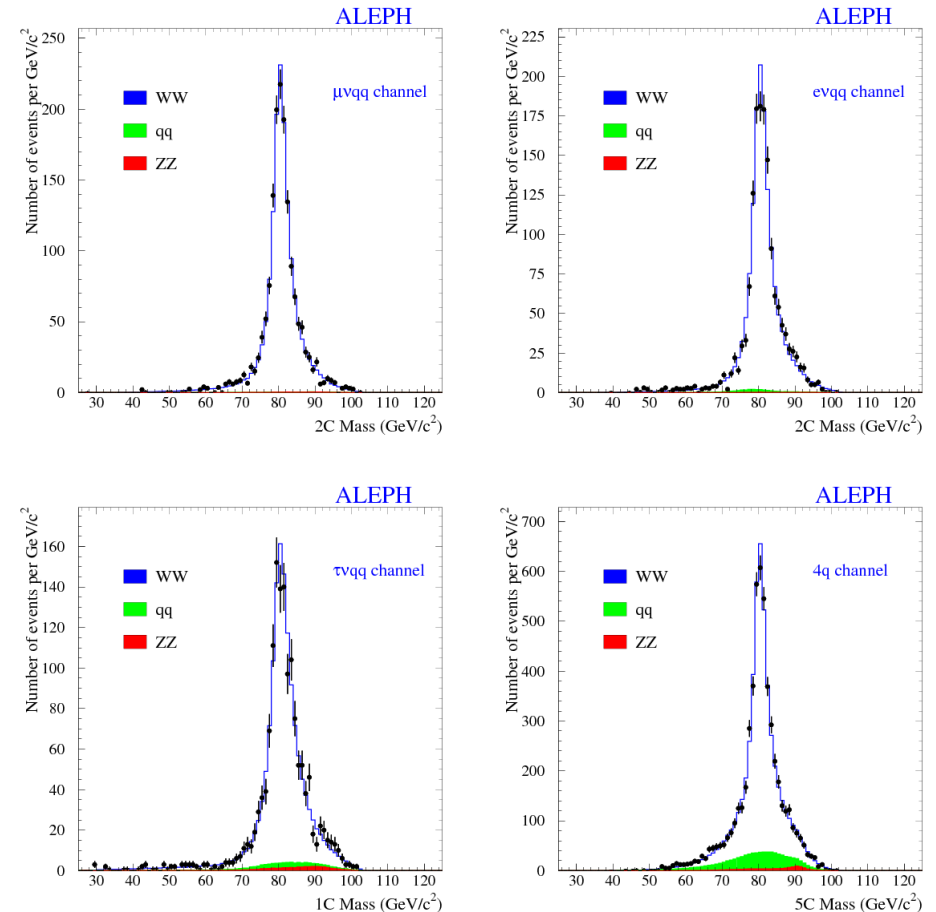
$$M_Z^2 = s \frac{\beta_1 \sin \theta_1 + \beta_2 \sin \theta_2 - \beta_1 \beta_2 |\sin(\theta_1 + \theta_2)|}{\beta_1 \sin \theta_1 + \beta_2 \sin \theta_2 + \beta_1 \beta_2 |\sin(\theta_1 + \theta_2)|}$$

E_{CM} is again a main ingredient: sets jet energy scale
 other main ingredients are the jets (and lepton) **angles**
 secondary ingredients are the **jet velocities** ($\beta = p/E$)

statistical uncertainties ALEPH LEP2 \rightarrow FCCee extrapolated

Stat uncertainty	Δm_W	$\Delta \Gamma_W$
evqq	87 MeV \rightarrow 0.9 MeV	200 MeV \rightarrow 2 MeV
$\mu\nu$ qq	82 MeV \rightarrow 0.8 MeV	200 MeV \rightarrow 2 MeV
$\tau\nu$ qq	121 MeV \rightarrow 1.2 MeV	320 MeV \rightarrow 3.2 MeV
qqqq	70 MeV \rightarrow 0.7 MeV	120 MeV \rightarrow 1.2 MeV
combined	43 MeV \rightarrow 0.4 MeV	90 MeV \rightarrow 0.9 MeV

LEP2 (ALEPH) from $\sim 10\text{k WW}$ @ $E_{\text{CM}} = 183\text{-}209$ GeV



W kinematic fit : systematics

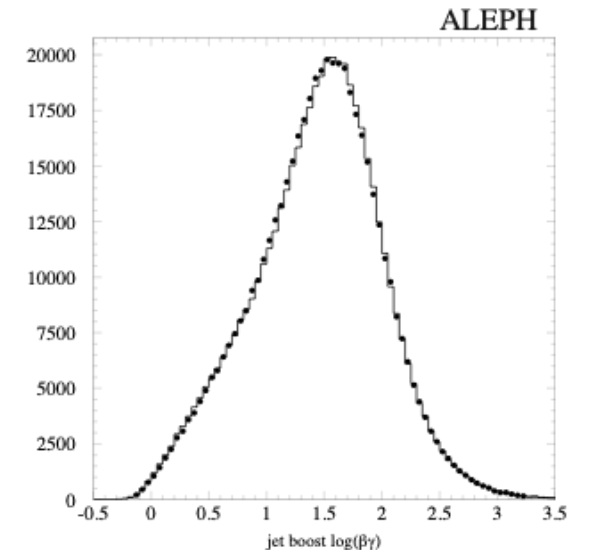
EPOL $\Delta E_{\text{CM}}=0.3$ MeV at $E_{\text{CM}}=162.6$ GeV
 [with Δm_W (stat)(162) ~ 1 MeV]

For larger E_{beam} at $E_{\text{CM}}=240-365$ GeV can make use of
 radiative Z-returns ($Z\gamma$) and ZZ events
 $\Delta E_{\text{CM}}(240\text{GeV})\sim 2$ MeV & $\Delta E_{\text{CM}}(365\text{ GeV})\sim 10$ MeV

Table 9: Summary of the systematic errors on m_W and Γ_W in the standard analysis averaged over 183-209 GeV for all semileptonic channels. The column labelled $\ell\nu q\bar{q}$ lists the uncertainties in m_W used in combining the semileptonic channels.

Source	Δm_W (MeV/ c^2)				$\Delta \Gamma_W$ (MeV)			
	$e\nu q\bar{q}$	$\mu\nu q\bar{q}$	$\tau\nu q\bar{q}$	$\ell\nu q\bar{q}$	$e\nu q\bar{q}$	$\mu\nu q\bar{q}$	$\tau\nu q\bar{q}$	$\ell\nu q\bar{q}$
e+ μ momentum	3	8	-	4	5	4	-	4
e+ μ momentum resoln	7	4	-	4	65	55	-	50
Jet energy scale/linearity	5	5	9	6	4	4	16	6
Jet energy resoln	4	2	8	4	20	18	36	22
Jet angle	5	5	4	5	2	2	3	2
Jet angle resoln	3	2	3	3	3	7	8	7
Jet boost	17	17	20	17	3	3	3	3
Fragmentation	10	10	15	11	22	23	37	25
Radiative corrections	3	2	3	3	3	2	2	2
LEP energy	9	9	10	9	7	7	10	8
Calibration ($e\nu q\bar{q}$ only)	10	-	-	4	20	-	-	9
Ref MC Statistics	3	3	5	2	7	7	10	5
Bkgnd contamination	3	1	6	2	5	4	19	7

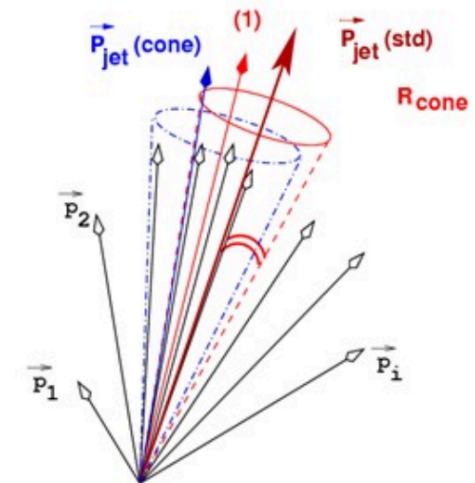
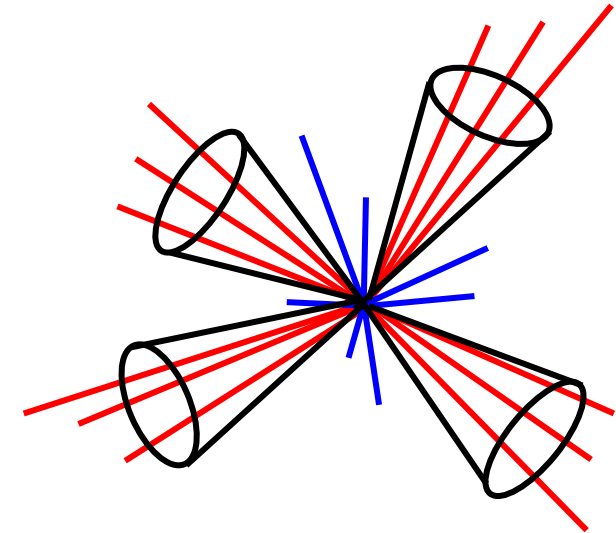
lepton and jet uncertainties
 from (Z) calibration data



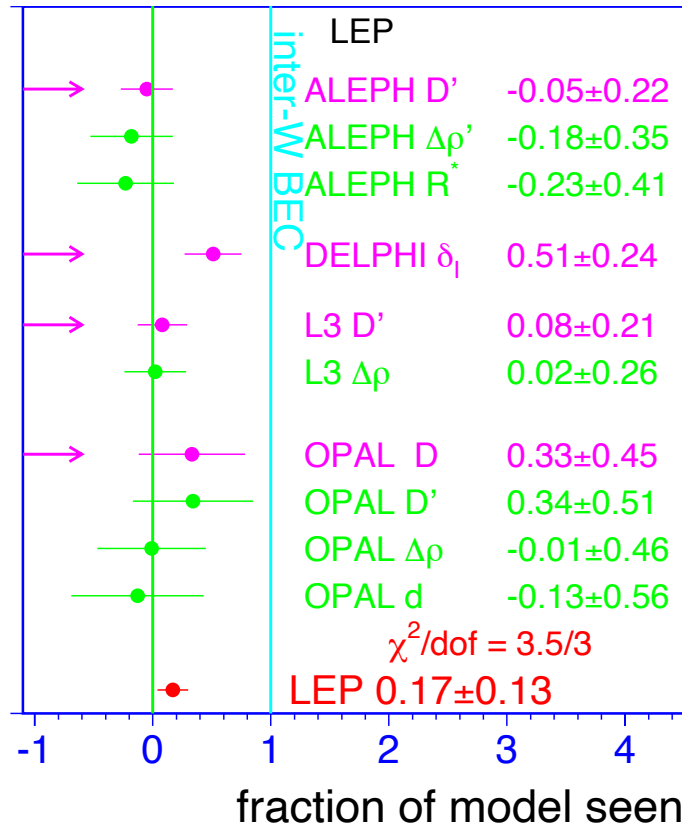
W kinematic fit : systematics in 4q

Table 8: Summary of the systematic errors on m_W and Γ_W averaged over 183-209 GeV in the $q\bar{q}q\bar{q}$ channel for the standard, PCUT (= 3.0 GeV/c) and CONE (R=0.4) reconstructions.

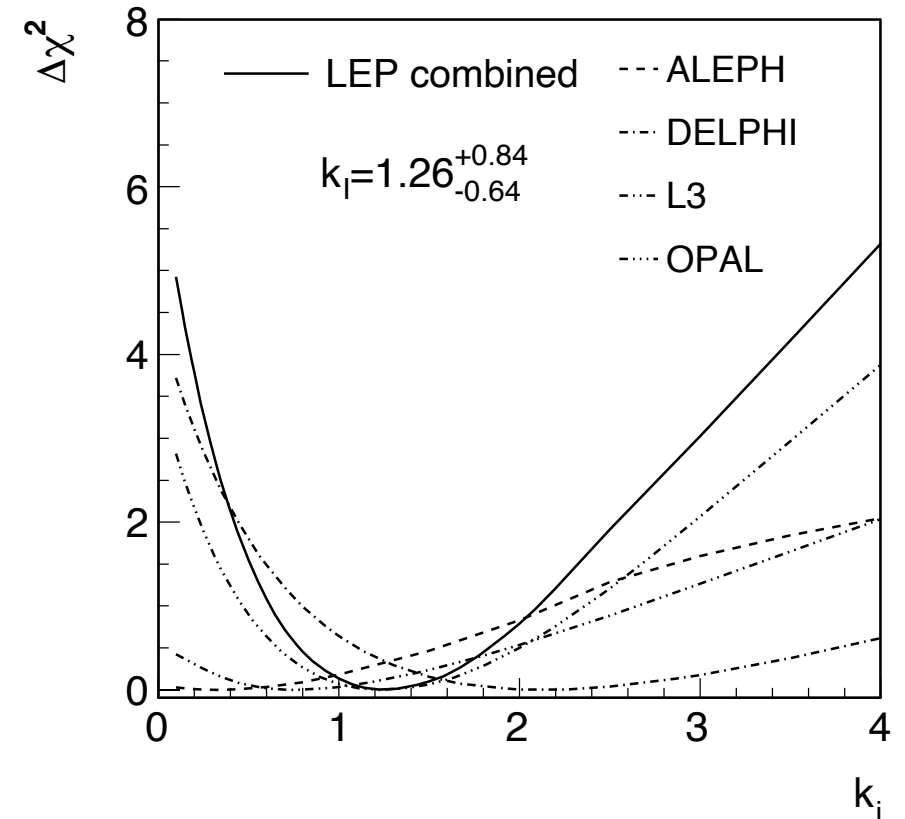
Source	Δm_W (MeV/c ²)			$\Delta \Gamma_W$ (MeV)		
	standard	PCUT	CONE	standard	PCUT	CONE
Jet energy scale/linearity	2	2	3	2	12	4
Jet energy resolu	0	1	0	7	9	10
Jet angle	6	6	6	1	3	3
Jet angle resolu	1	3	2	15	18	9
Jet boost	14	15	11	5	5	4
Fragmentation	10	20	20	20	40	40
Radiative Corrections	2	2	2	5	7	7
LEP energy	9	10	10	7	7	7
Ref MC Statistics	2	3	3	5	7	7
Bkgnd contamination	8	5	5	20	31	32
Colour reconnection	79	28	36	104	24	45
Bose-Einstein effects	0	2	3	20	10	10



W kinematic fit : FSI systematics in 4q



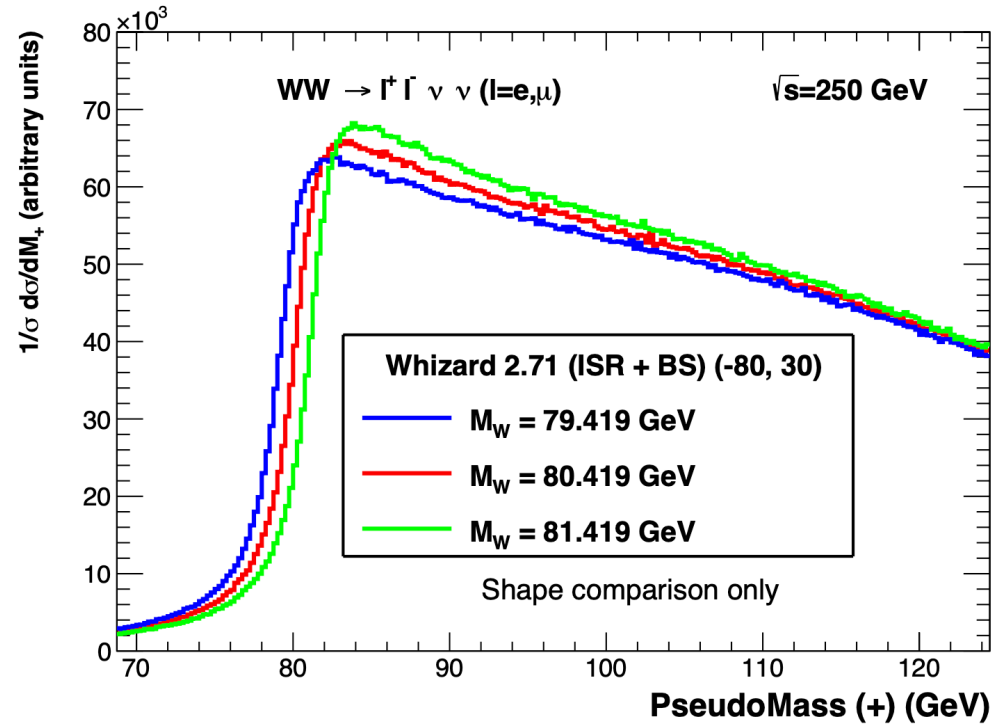
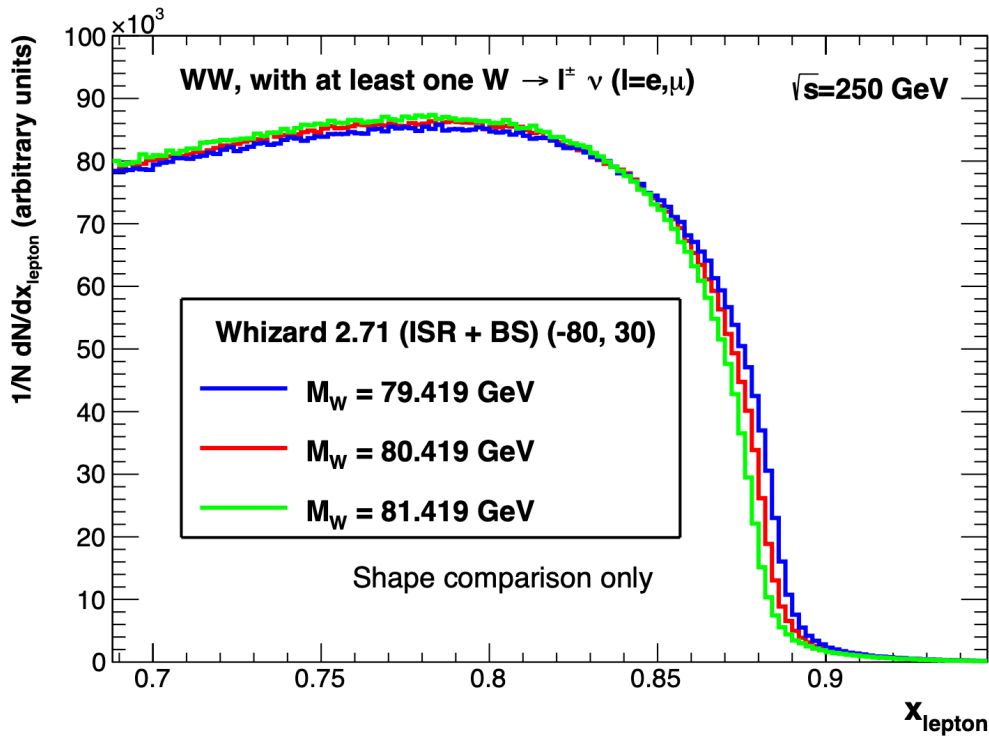
Measured BEC expressed as the relative fraction of the model with inter-W correlations.



Individual and LEP combined $\Delta\chi^2$ curves for the measurement of the CR parameter k_l in the SK1 model.

W mass from lepton Energy and Pseudomass

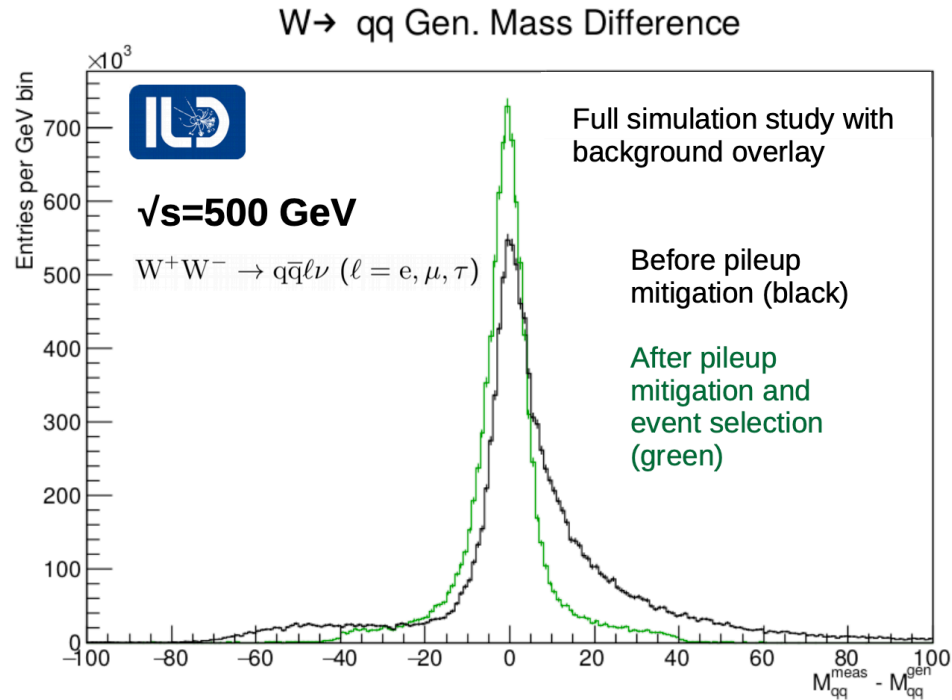
Endpoints in the lepton (or jet) energy a
 $E_\ell = E_{CM}(1 \pm \beta)$ where β is the W velocity



expected statistical $\Delta m_W = 4.4$ MeV with $2/ab@250$ GeV
 experimental syst from lepton energy calibration

W mass from the hadronic mass

[arXiv:2011.12451](https://arxiv.org/abs/2011.12451)



ΔM_W [MeV]	ILC	ILC	ILC	ILC
\sqrt{s} [GeV]	250	350	500	1000
\mathcal{L} [fb^{-1}]	500	350	1000	2000
$P(e^-)$ [%]	80	80	80	80
$P(e^+)$ [%]	30	30	30	30
jet energy scale	3.0	3.0	3.0	3.0
hadronization	1.5	1.5	1.5	1.5
pileup	0.5	0.7	1.0	2.0
total systematics	3.4	3.4	3.5	3.9
statistical	1.5	1.5	1.0	0.5
total	3.7	3.7	3.6	3.9

«.. dominated by the systematic uncertainties from the effective **jet energy scale** which is a challenging demand.. »

$$\Delta m_W = 0.3-0.4 \text{ MeV} \quad \Delta \Gamma_W = 1 \text{ MeV}$$

work ahead : WW threshold

- Evaluate theory requirements on total cross sections in the 157-162 GeV range
 - theory uncertainty evolution and correlation , 4f-interference effects
- Explore in more detail the **systematic uncertainties (cancellation) effects with multi-point ($n \geq 3$) cross section measurements**. Evaluate benefits of additional model independence.
 - reduction / cancellation of **acceptance & luminosity systs** is of particular interest
- Design a realistic a modern analysis with event classifiers, evaluate performances and the corresponding **impact of systematic uncertainties**. Feedback to theory and detector design.
-

work ahead : W decay kinematics 1

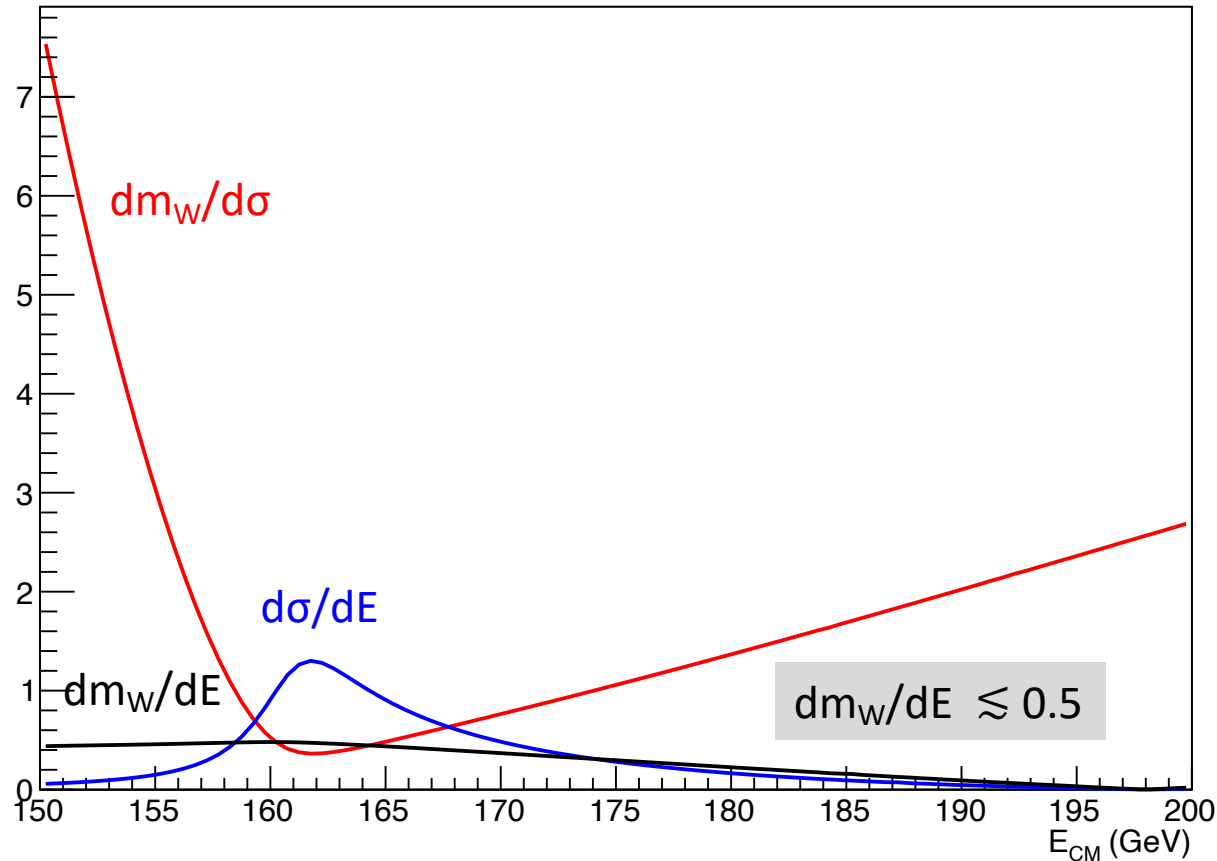
- Studies with a LEP-style m_W measurement : verify stat potential with different E_{CM} data and study the **impact of systematic uncertainties in detail** : feedback to theory and detector design
- Ultimate **simultaneous analysis and fit** of diboson events (WW, ZZ and $Z\gamma$) to extract m_W/m_Z with potential cancellations of systematic uncertainties both theoretical and experimental
- ...

work ahead : W decay kinematics 2

- kinematic reconstruction methods that do not make use of E_{CM} . Most demanding on experimental systs (energy & momentum calibration of jets and leptons) . \rightarrow Detector requirements
- ...
- dedicated discussion on 4-jet final state interconnection effects
 - different impact of effects with or without E_{cm} kin fits ?
 - what will be the impact of CR (BE) effects ? Can it be avoided/reduced with dedicated strategies (pcut, cone, ...)
 - How will CR (BE) be measured/constrained in situ (inter-jet WW- \rightarrow 4jets activity) and in other hadronic final states eg Z- \rightarrow multijets . Viable models ?

backup

The WW threshold W mass : beam energy



$$\Delta m_W(E) = \left(\frac{d\sigma}{dm_W} \right)^{-1} \left(\frac{d\sigma}{dE} \right) \Delta E \leq \frac{1}{2} \Delta E$$

Uncertainty on beam energy $\Delta E_b = \frac{1}{2} \Delta E$
 translates directly to m_W

$$\Delta E_b \cong \Delta m_W$$

Very limited variations of the dm_W/dE coefficient with E_{CM} in the threshold region

WW threshold : W mass precision requirements

Conditions to achieve $\Delta m_W(\text{syst}) < \Delta m_W(\text{stat}) = \mathbf{0.3 \text{ MeV}}$
with a single point WW threshold measurement

current theory precision
 $\Rightarrow \Delta m_W = 3 \text{ MeV}$

$$\Delta m_W(B) = \left(\frac{d\sigma}{dm_W} \right)^{-1} \left(\frac{\Delta\sigma_B}{\varepsilon} \oplus \Delta\sigma_{TH} \right)$$

Background and Theory

$$\Delta\sigma_{TH} < \mathbf{1 \text{ fb}} \quad (\Delta\sigma_{TH}/\sigma_{TH} < 2 \cdot 10^{-4})$$

$$\Delta\sigma_B/\varepsilon < \mathbf{1 \text{ fb}} \quad (\Delta\sigma_B/\sigma_B < 4 \cdot 10^{-3})$$

$$\Delta m_W(\varepsilon) = \sigma \left(\frac{d\sigma}{dm_W} \right)^{-1} \left(\frac{\Delta\varepsilon}{\varepsilon} + \frac{\Delta L}{L} \right)$$

Acceptance and Luminosity

$$\left(\frac{\Delta\varepsilon}{\varepsilon} \oplus \frac{\Delta L}{L} \right) < \mathbf{2 \cdot 10^{-4}}$$

$$\Delta m_W(E) = \left(\frac{d\sigma}{dm_W} \right)^{-1} \left(\frac{d\sigma}{dE} \right) \Delta E \leq \frac{1}{2} \Delta E$$

Collision energy

$$\Delta E_b < \mathbf{0.3 \text{ MeV}} \quad (\Delta E_b/E_b < 4 \cdot 10^{-6})$$

The WW threshold : background syst

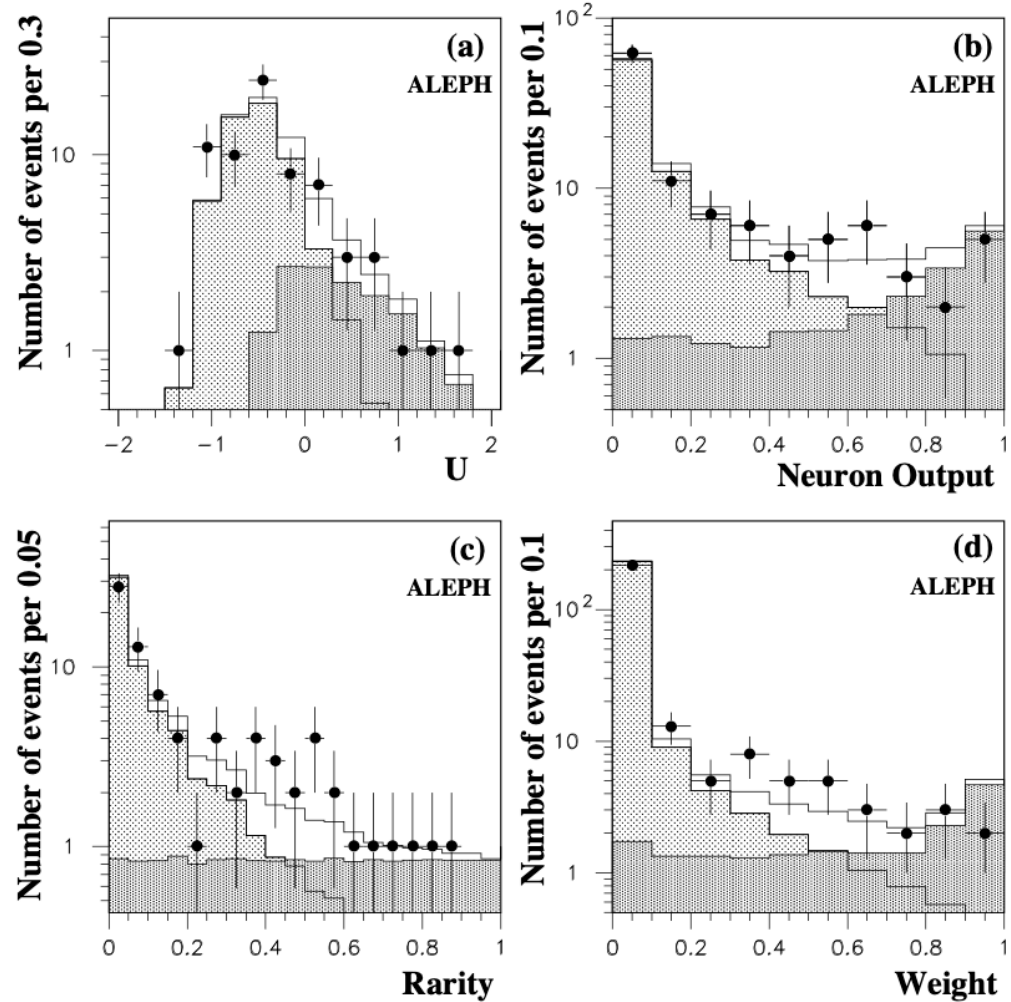
almost all bkg in the 4q channel

Selection	Expected signal	Expected background	Observed
$W^+W^- \rightarrow q\bar{q}q\bar{q}$	9.6 ± 1.0	3.44 ± 0.39	14
$W^+W^- \rightarrow q\bar{q}e\bar{\nu}_e$	3.89 ± 0.44	0.18 ± 0.27	3
$W^+W^- \rightarrow q\bar{q}\mu\bar{\nu}_\mu$	4.19 ± 0.46	0.27 ± 0.15	2
$W^+W^- \rightarrow q\bar{q}\tau\bar{\nu}_\tau$	2.32 ± 0.28	0.96 ± 0.34	7
$W^+W^- \rightarrow \ell^+\nu_\ell\ell'^-\bar{\nu}_{\ell'}$	2.58 ± 0.28	$0.19^{+0.12}_{-0.04}$	2
Combined	22.6 ± 2.4	5.0 ± 0.6	28

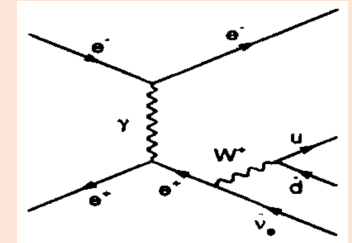
OPAL Phys. Lett. B 389 (1996) 416.

[Phys.Lett.B 401 \(1997\) 347](#)

purity ~95% achieved in the last bins



4-fermion-CC03
interference effects



positive & negative
effects (10-50 fb)
reported in the various
channels, within the LEP
analyses acceptance

WW threshold : acceptance syst

Syst unc at higher E_{CM} (207 GeV) on σ_{WW} ($\sim 16\text{pb}$)

Source	uncertainty (fb)			
	$l\nu l\nu$	$l\nu q\bar{q}$	$q\bar{q}q\bar{q}$	total
Tracking	4	19	31	50
Simulation of calorimeters	-	9	26	31
Hadronization models	-	27	8	35
Z peak $q\bar{q}$ fragmentation	-	-	20	20
inter-W final state interaction	-	-	28	28
Background contamination	9	5	31	35
Lepton identification	1	2	-	3
Beam-related background	10	17	37	22
$\mathcal{O}(\alpha)$ corrections DPA	2	9	12	6
Luminosity	8	35	44	87
Simulation statistics	6	20	14	25
Total	17	57	87	126

ALEPH [Eur.Phys.J.C 38 \(2004\) 147](#)

Source	$\sigma_{\text{WW}}^{q\bar{q}q\bar{q}}$ (pb)	$\sigma_{\text{WW}}^{q\bar{q}l\nu}$ (pb)	$\sigma_{\text{WW}}^{l\nu l\nu}$ (pb)
Four-jet modelling	± 0.051	± 0.014	-
Background cross-sections	$+0.009$	$+0.016$	± 0.006
Fragmentation	± 0.045	± 0.038	-
Final state interactions	± 0.025	-	-
Radiative corrections	± 0.008	± 0.008	± 0.002
Luminosity (theor)	± 0.011	± 0.010	± 0.002
Luminosity (exp)	± 0.045	± 0.043	± 0.011
Detector effects	± 0.045	± 0.053	± 0.033
Monte Carlo statistics	± 0.005	± 0.014	± 0.033

DELPHI [Eur.Phys.J.C 34 \(2004\) 127](#)

can roughly scale/4 for equivalent ϵ effects at threshold σ_{WW} ($\sim 4\text{pb}$)

target : bring table items below $4\text{fb}/4=1\text{fb}$

20-30fb on tables $\Rightarrow \Delta m_W = 1.5\text{-}2\text{ MeV}$

NP QCD effects have important impacts on both $q\bar{q}q\bar{q}$ and $q\bar{q}l\nu$

need improvements in fragmentation and hadronization modeling plus constraints from control data ($Z \rightarrow q\bar{q}$)

less worrisome than using jet properties for kin reco

WW threshold @ ILC

[arXiv:1603.06016](https://arxiv.org/abs/1603.06016) & [arXiv:1908.11299](https://arxiv.org/abs/1908.11299)

ILC polarised collisions : enhance (x4) t-channel
WW production or suppress it to control background

Channel	Efficiency (%)	σ_{bkgd}^U (fb)	A_{LR}^B	Eff. syst. (%)	Bkgd syst.	A_{LR}^B syst.
lvlv	87.5	10	0.15	0.1	free	0.025
qqlv	87.5	40	0.30	0.1	free	0.012
qqqq	83.5	200	0.48	0.1	free	0.005

Table 3: Experimental assumptions for the WW event selection near threshold using a polarized scan

with 100 fb⁻¹

Fit type	Uncertainty source	ΔM_W [MeV]	ΔM_W (syst.) [MeV]
fixbkg	Background	3.20	2.30
fixpol	Polarization	3.73	1.27
fixeff	Efficiency	3.86	1.18
fixlum	Luminosity	3.76	0.78
fixALRB	A_{LR}^B	3.86	0.80
fixall	Statistical	2.43	
	Systematic		3.10
standard	Total Error	3.94	

$$\Delta m_W (\text{MeV}) = 2.4 (\text{stat}) \oplus 3.1 (\text{syst}) \oplus 0.8 (\sqrt{s}) \oplus \text{theory}$$

fitted $\Delta\varepsilon \sim 10^{-3}$ and $\Delta\sigma_B \sim 6$ fb
additional impact of pol uncertainty

\sqrt{s} (GeV)	L (fb ⁻¹)	f	$\lambda_e - \lambda_{e^+}$	N_{ll}	N_{lh}	N_{hh}	N_{RR}
160.6	4.348	0.7789	--+	2752	11279	12321	926968
		0.1704	+--	20	67	158	139932
		0.0254	++	2	19	27	6661
		0.0254	--	21	100	102	8455
161.2	21.739	0.7789	--+	16096	67610	73538	4635245
		0.1704	+--	98	354	820	697141
		0.0254	++	37	134	130	33202
		0.0254	--	145	574	622	42832
161.4	21.739	0.7789	--+	17334	72012	77991	4639495
		0.1704	+--	100	376	770	697459
		0.0254	++	28	104	133	33556
		0.0254	--	135	553	661	42979
161.6	21.739	0.7789	--+	18364	76393	82169	4636591
		0.1704	+--	81	369	803	697851
		0.0254	++	43	135	174	33271
		0.0254	--	146	618	681	42689
162.2	4.348	0.7789	--+	4159	17814	19145	927793
		0.1704	+--	16	62	173	138837
		0.0254	++	10	28	43	6633
		0.0254	--	46	135	141	8463
170.0	26.087	0.7789	--+	63621	264869	270577	5560286
		0.1704	+--	244	957	1447	838233
		0.0254	++	106	451	466	40196
		0.0254	--	508	2215	2282	50979

Table 1: Illustrative example of the numbers of events in each channel for the standard 100 fb⁻¹ 6-point ILC scan with 4 helicity configurations. Columns give the center-of-mass energy, \sqrt{s} , the apportioned integrated luminosity, the fraction for each helicity configuration, $\lambda_e - \lambda_{e^+}$, and the numbers of events observed in each channel.

WW threshold : W mass and width

With cross section $\sigma_1 \sigma_2$ measurements at two energies $E_1 E_2$: uncertainty propagation

$$\begin{cases} \sigma_1 = \sigma_{WW}(E_1, m_W, \Gamma_W) \\ \sigma_2 = \sigma_{WW}(E_2, m_W, \Gamma_W) \end{cases} \quad \begin{cases} \Delta\sigma_1 = a_1 \Delta m + b_1 \Delta\Gamma \\ \Delta\sigma_2 = a_2 \Delta m + b_2 \Delta\Gamma \end{cases} \quad \begin{matrix} a_1 = \frac{d\sigma_1}{dm} & b_1 = \frac{d\sigma_1}{d\Gamma} \\ a_2 = \frac{d\sigma_2}{dm} & b_2 = \frac{d\sigma_2}{d\Gamma} \end{matrix}$$

$$\Delta m = - \frac{b_2 \Delta\sigma_1 - b_1 \Delta\sigma_2}{a_2 b_1 - a_1 b_2}$$

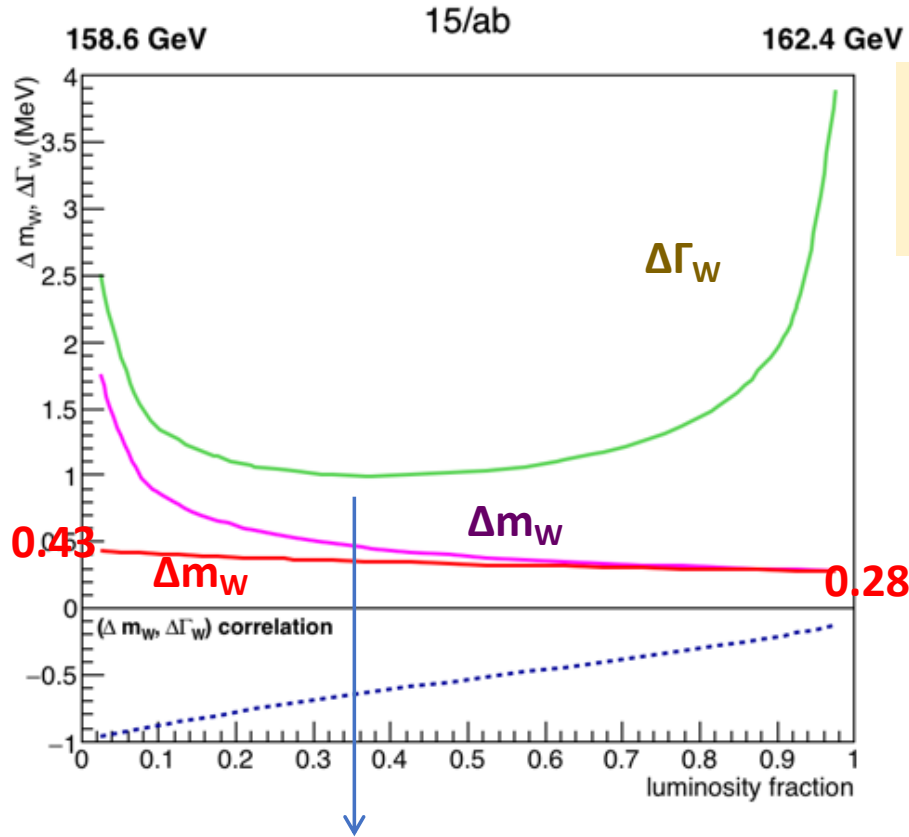
$$\Delta\Gamma = \frac{a_2 \Delta\sigma_1 - a_1 \Delta\sigma_2}{a_2 b_1 - a_1 b_2}$$

$\Delta m, \Delta\Gamma$ linear correlation with uncorrelated $\Delta\sigma_1, \Delta\sigma_2$

$$r = - \frac{1}{\Delta m \Delta\Gamma} \frac{a_2 b_2 \Delta\sigma_1^2 + a_1 b_1 \Delta\sigma_2^2}{(a_2 b_1 - a_1 b_2)^2}$$

WW threshold : W mass and width

Scans of possible E_1 E_2 data taking energies and luminosity fractions f (at the E_2 point)



$\Delta m_W = 0.45$ MeV , $\Delta \Gamma_W = 1$ MeV ($r = -0.6$)
 $\Delta m_W = 0.35$ MeV

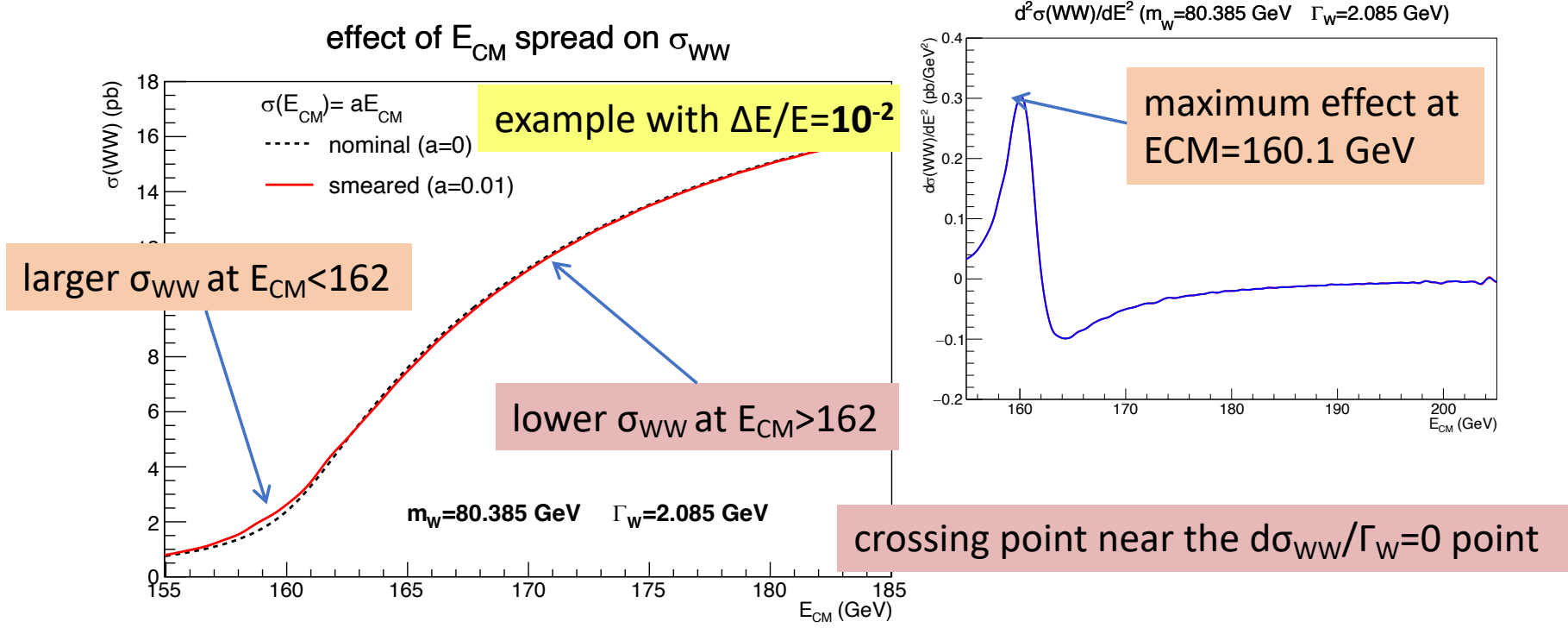
A - minimum of $\Delta \Gamma_W = 0.91$ MeV with $\Delta m_W = 0.55$ MeV
 taking data at $E_1 = 156.6$ GeV $E_2 = 162.4$ GeV $f = 0.25$
 yields $\Delta m_W = 0.47$ MeV (as single par)

B- minimum of $\Delta m_W = 0.28$ MeV $\Delta \Gamma_W = 3.3$ MeV with
 $E_1 = 155.5$ GeV $E_2 = 162.4$ GeV $f = 0.95$
 yields $\Delta m_W = 0.28$ MeV (as single par)

C- minimum of $\Delta \Gamma_W = 0.96$ MeV + $\Delta m_W = 0.41$ MeV with
 $E_1 = 157.5$ GeV $E_2 = 162.4$ GeV $f = 0.45$
 yields and $\Delta m_W = 0.37$ MeV (as single par)

$\Delta m_W, \Delta \Gamma_W$: error on W mass and width from fitting both
 Δm_W : error on W mass from fitting only m_W

WW threshold : energy spread effects



$\sigma(E_{CM}) = (0.47 - 1.10) \cdot 10^{-3} E_{CM}$

Optimal m_W & Γ_W points @ $E_{CM} = 157.3$ & 162.6 GeV
 $\rightarrow \Delta\sigma_{WW} = +(0.24 - 1.3)$ fb & $-(0.18 - 1.0)$ fb
 $\rightarrow \Delta m_W = -(0.09 - 0.48)$ MeV
 $\rightarrow \Delta \Gamma_W = +(0.6 - 3.3)$ MeV

Maximum effects are at the level of $\Delta m_W(\text{stat})$ and $2x \Delta \Gamma_W(\text{stat})$ so that control on the beam energy RMS < 50% is required to avoid additional syst contributions from this source