

META2.0 parton distributions

Overview + details in the backup



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Introduction

I will present version 2 of meta-parametrizations of PDFs for combination of PDF+ α_s uncertainties from PDF ensembles of CT14, MMHT'14, and NNPDF3.0.

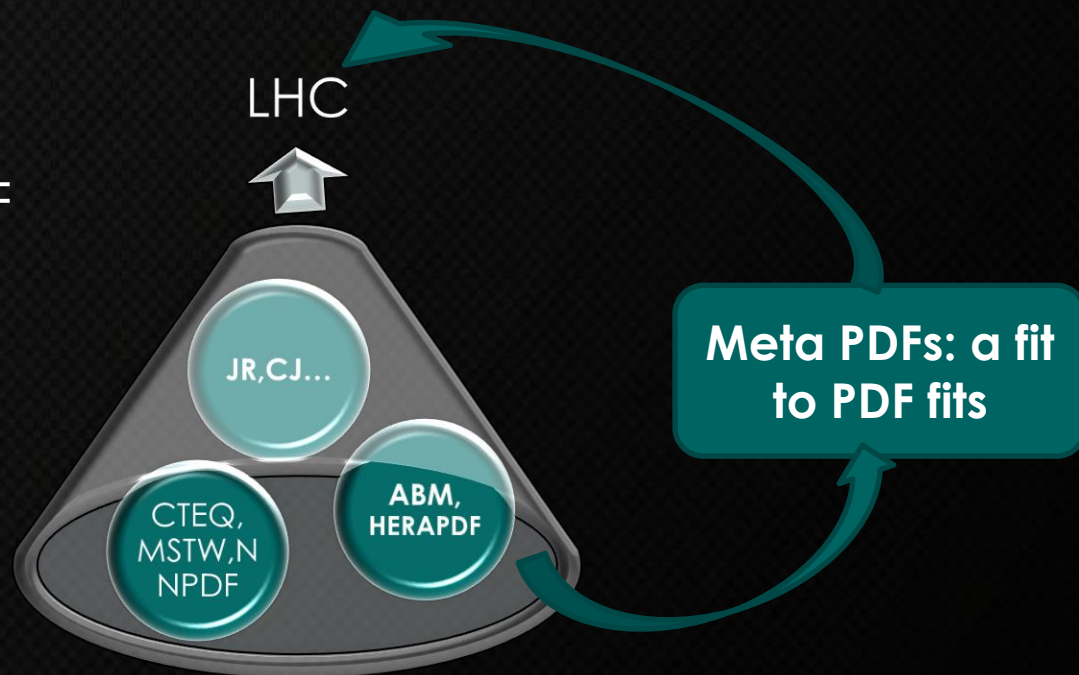
The new version made several advances compared to META1.0 ensemble published in *arXiv:1401.0013*. It utilizes an advanced parametrization form, reproduces PDF uncertainties and correlations of all input PDFs with 40-60 Hessian error sets, and provides a method to compute asymmetric errors.

A public Mathematica module MP4LHC for meta-analysis is available for beta-testing

What is the PDF meta-analysis?

A meta-analysis **compares** and **combines** LHC predictions based on several PDF ensembles. It serves the same purpose as the PDF4LHC prescription. It combines the PDFs directly in space of PDF parameters. It can significantly reduce the number of error PDF sets needed for computing PDF uncertainties and PDF-induced correlations.

The number of input PDF ensembles that can be combined is almost unlimited



META 1.0 PDFs: A working example of a meta-analysis

See arXiv:1401.0013 for details

1. Select the input PDF ensembles (CT, MSTW, NNPDF...)
2. Fit each PDF error set in the input ensembles by a common functional form ("**a meta-parametrization**")
3. Generate many Monte-Carlo replicas from meta-parametrizations of each set to investigate the probability distribution on the ensemble of all meta-parametrizations (as in Thorne, Watt, 1205.4024)
4. Construct a final ensemble of 68% c.l. **Hessian eigenvector sets** to propagate the PDF uncertainty from the combined ensemble of replicated meta-parametrizations into LHC predictions.



parameter,

as defined by Merriam-Webster dictionary

...

- an arbitrary constant whose value characterizes a member of a system **(as a family of curves)**
- any of a set of physical properties whose values determine the characteristics or behavior of something <parameters of the atmosphere such as temperature, pressure, and density>

META parameters of PDFs

- The core idea of the meta-analysis approach is to cast all input PDFs into a shared parametric representation.
- META parameters can be selected in many ways
 - By fitting $f_i(x, Q)$ by flexible functions $F_i(\{a\}; x, Q)$, such as those based on Bernstein polynomials (*our approach*)
 - By treating the PDF values themselves as parameters, $f_i(x_j, Q_l) \equiv f_{ijl}$ (Carrazza et al., 1505.06736)

...

Conjecture

- Method 2 is really a variation of method 1
 - Step functions in bins of x and Q are employed instead of continuous functions
 - Minimization is performed with one of genetic algorithms rather than with traditional analytic minimization

Does the META parametrization introduce a bias?

- Yes. But we can demonstrate that the bias is negligible with reasonable choices.
- Even the “unbiased” re-parametrization utilizing PDF values is biased by choices related to sampling of x and Q grids, selection of genetic algorithm, quantities to minimize, etc.

In Eq. (3) we have introduced a sampling in x , with a total of N_x points. This immediately raises the issue of choosing both a suitable spacing and range of the grid of points in x . Because PDFs are generally quite smooth, neighboring points in x are highly correlated, and thus the x -grid cannot be too fine-grained, otherwise the matrix cov^{pdf} rapidly becomes ill-conditioned. Furthermore, the choice of the x -grid range must keep into account not only the fact that we want the replicas to be especially well-reproduced where they are accurately known (hence the grid should not be dominated by points in extrapolation regions), but also that the whole procedure is meaningful only if the starting probability distribution is at least approximately Gaussian. The way both issues are handled will be discussed in detail in Sect. 2.2 below.

From 1505.06736, page 5

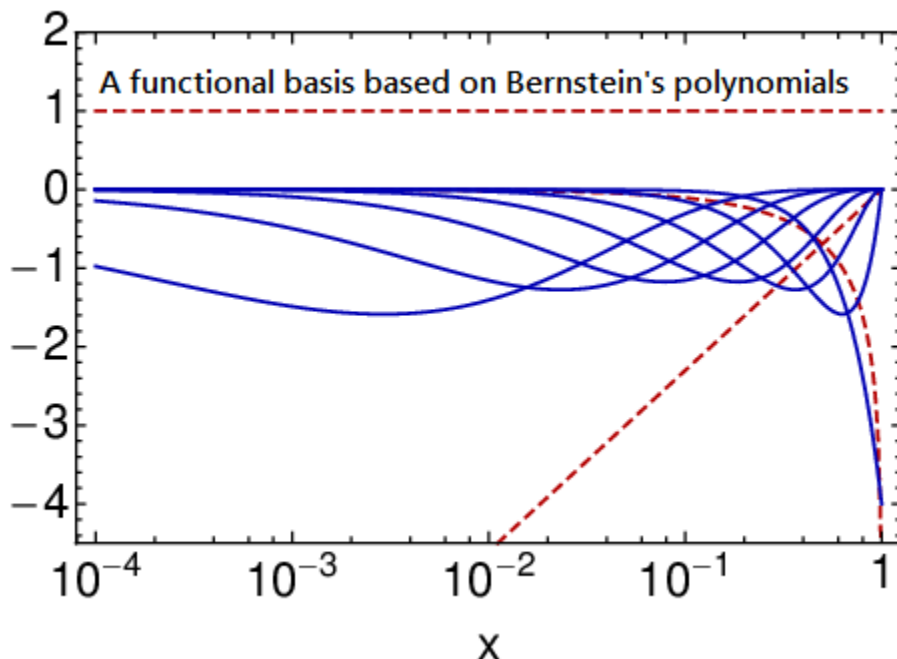
META PDFs: functional forms

New

v. 1.0: Chebyshev polynomials (Pumplin, 0909.5176, Glazov, et al., 1009.6170, Martin, et al., 1211.1215)

v 2.0: Bernstein polynomials \Rightarrow more faithful reproduction of the full ensemble of MC replicas. (Pumplin) Peaks occur at different x , reducing correlations between PDF parameters.

The initial scale of DGLAP evolution is $Q_0=8 \text{ GeV}$.



The meta-parametrizations are fitted to the input PDFs at $x > 3 \cdot 10^{-5}$ for all flavors ; $x < 0.4$ for \bar{u}, \bar{d} ; $x < 0.3$ for s, \bar{s} ; and $x < 0.8$ for other flavors. PDFs outside these x regions are determined entirely by extrapolation.

The logic behind the META approach

Emphasize simplicity and intuition

When expressed as the meta-parametrizations, PDF functions can be combined by averaging their meta-parameter values

Standard error propagation is more feasible, e.g., to treat the meta-parameters as discrete data in the linear (Gaussian) approximation for small variations

The Hessian analysis can be applied to the combination of all input ensembles in order to optimize uncertainties and eliminate “noise”

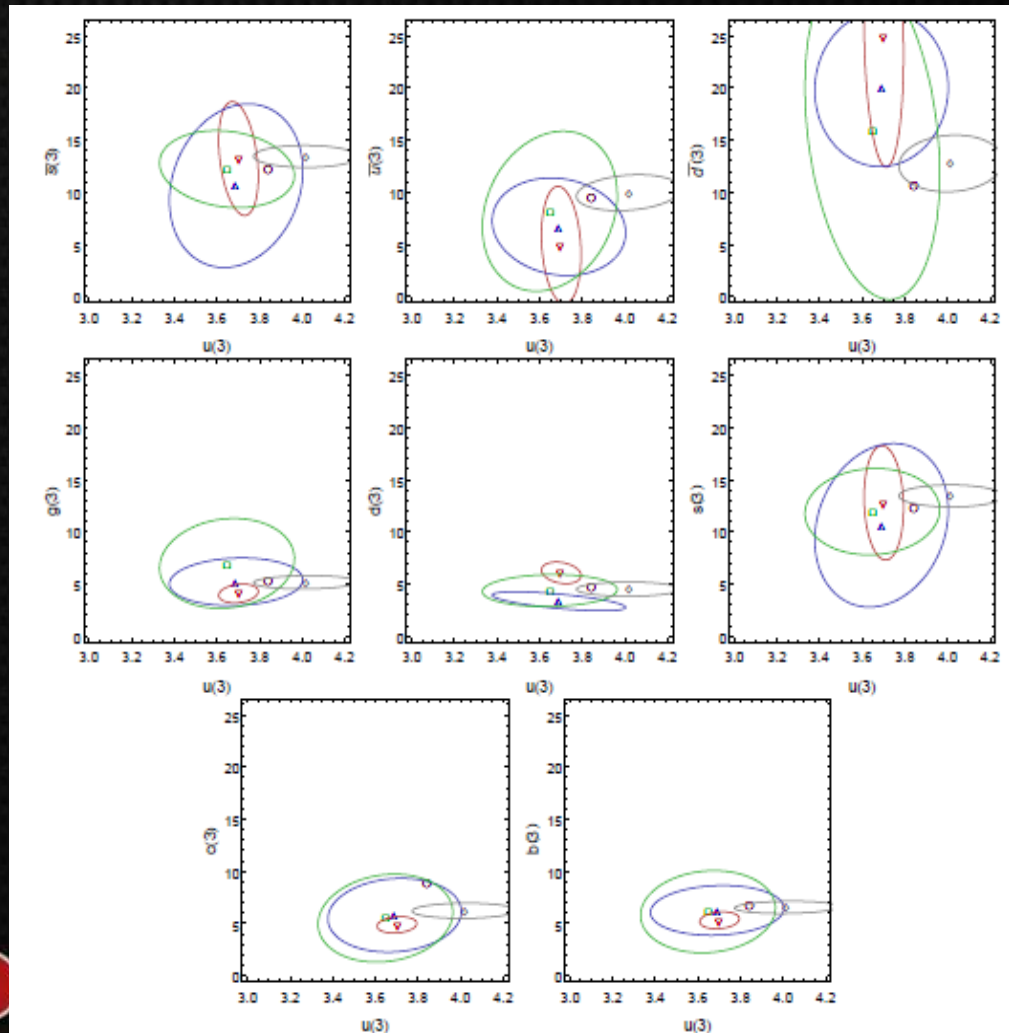
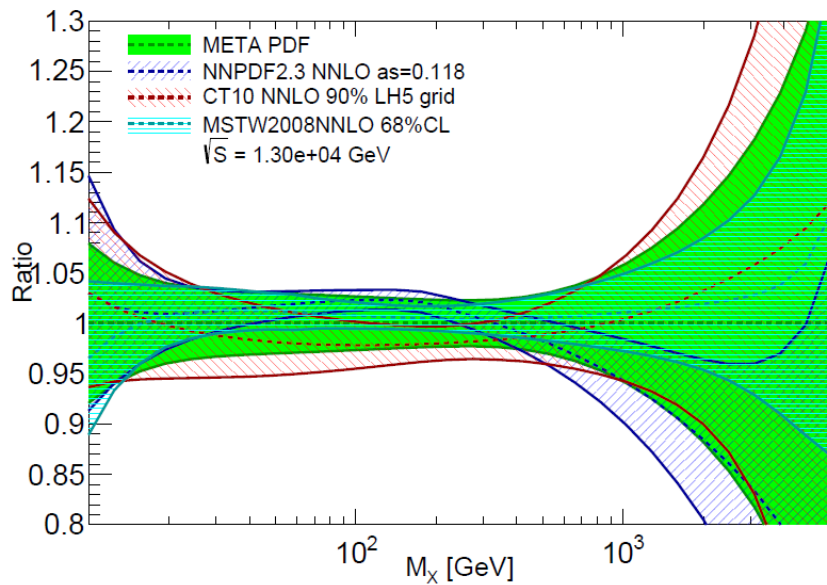


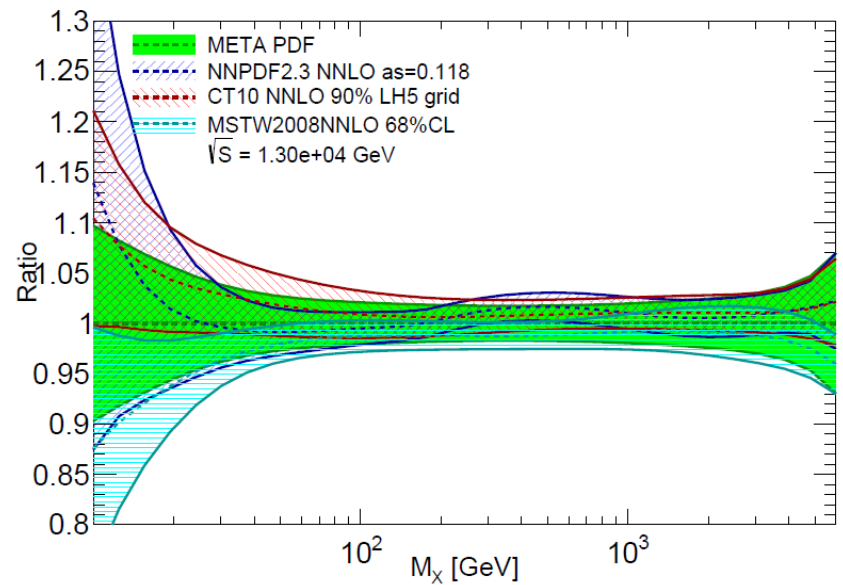
Figure 10: Fitted PDF parameters and 90% c.l. ellipses for CT10 (blue up triangle), MSTW08 (red down triangle), NNPDF2.3 (green square), HERAPDF1.5 (gray diamond) and ABM11 (magenta circle).

Some parton luminosities

Gluon-Gluon, luminosity

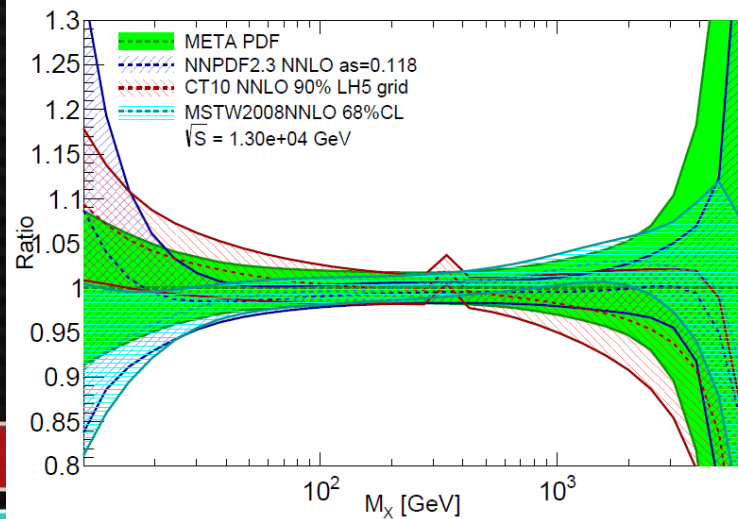


Quark-Quark, luminosity



Generated with APFEL 3.0.0 Web

Quark-Antiquark, luminosity



Generated with APFEL 3.0.0 Web

Plots are made with APFEL WEB (apfel.mi.infn.it; Carrazza et al., [1410.5456](https://arxiv.org/abs/1410.5456))

- More illustrations of the META approach are in backup slides.
- The META methodology is very flexible. Let's talk about our specific choices.

Reduction of the error PDFs

The number of final error PDFs is much smaller than in the input ensembles

In the META2.0 study:

208 CT'14, MMHT'14, NNPDF3.0 error sets

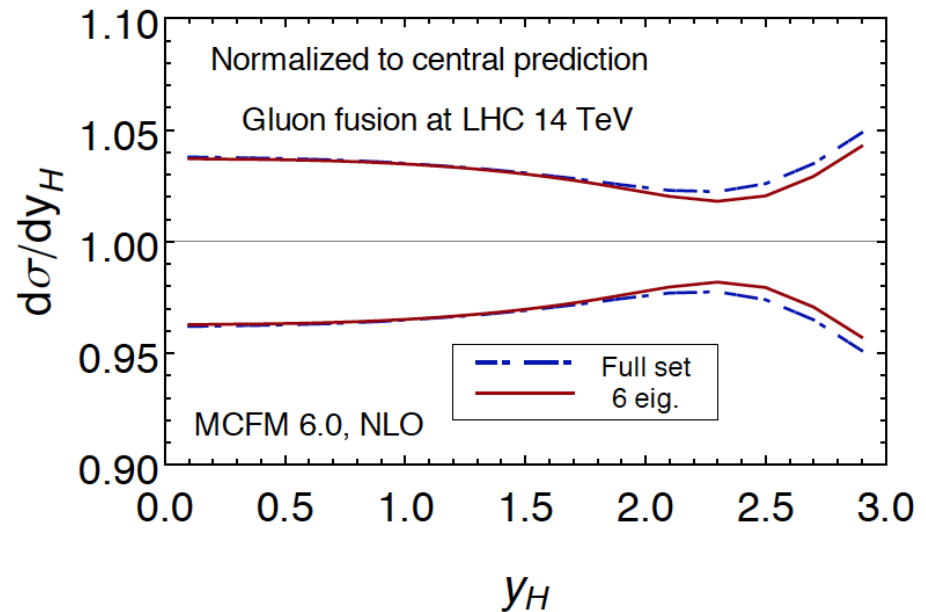
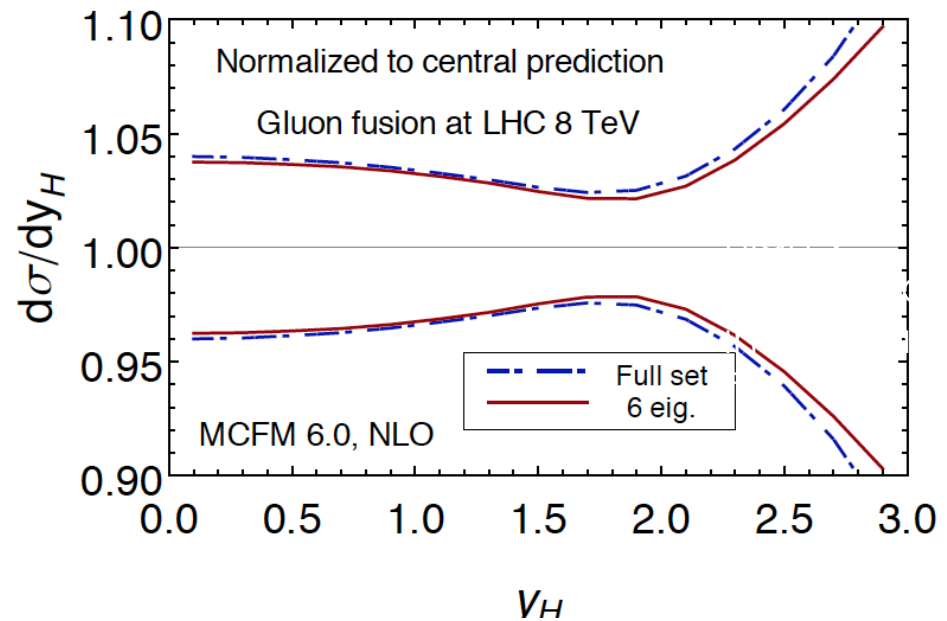
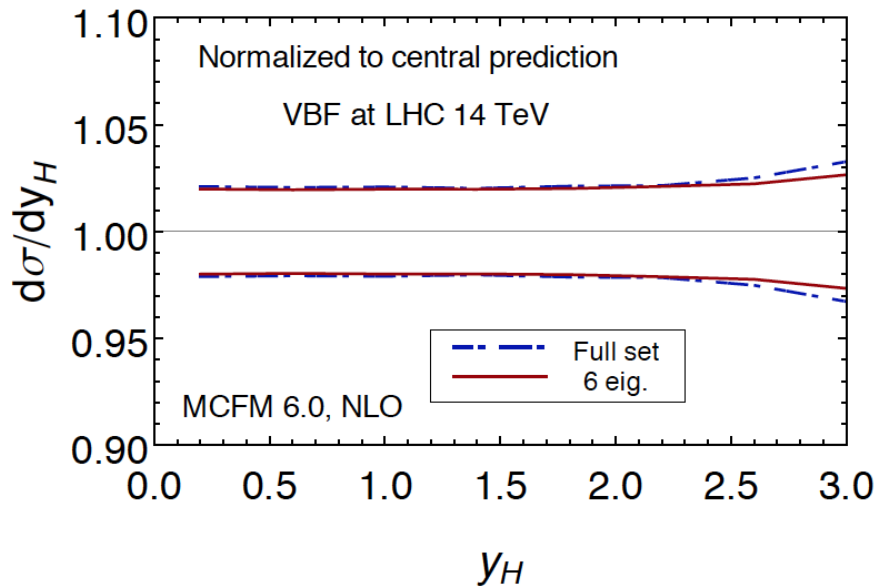
⇒ 600 MC replicas for reconstructing the combined probability distribution

⇒ 40, 60, 100 Hessian META sets for most LHC applications (**general-purpose** ensembles META2.0)

⇒ 13 META sets for LHC Higgs production observables (**reduced ensemble** META LHCH, obtained using the method of data set diagonalization)

Higgs eigenvector set

- The reduced META eigenvector set does a good job of describing the uncertainties of the full set for *typical* processes such as ggF or VBF
- But actually does a good job in reproducing PDF-induced correlations and describing those LHC physics processes in which g , \bar{u} , \bar{d} drive the PDF uncertainty (see next slide)



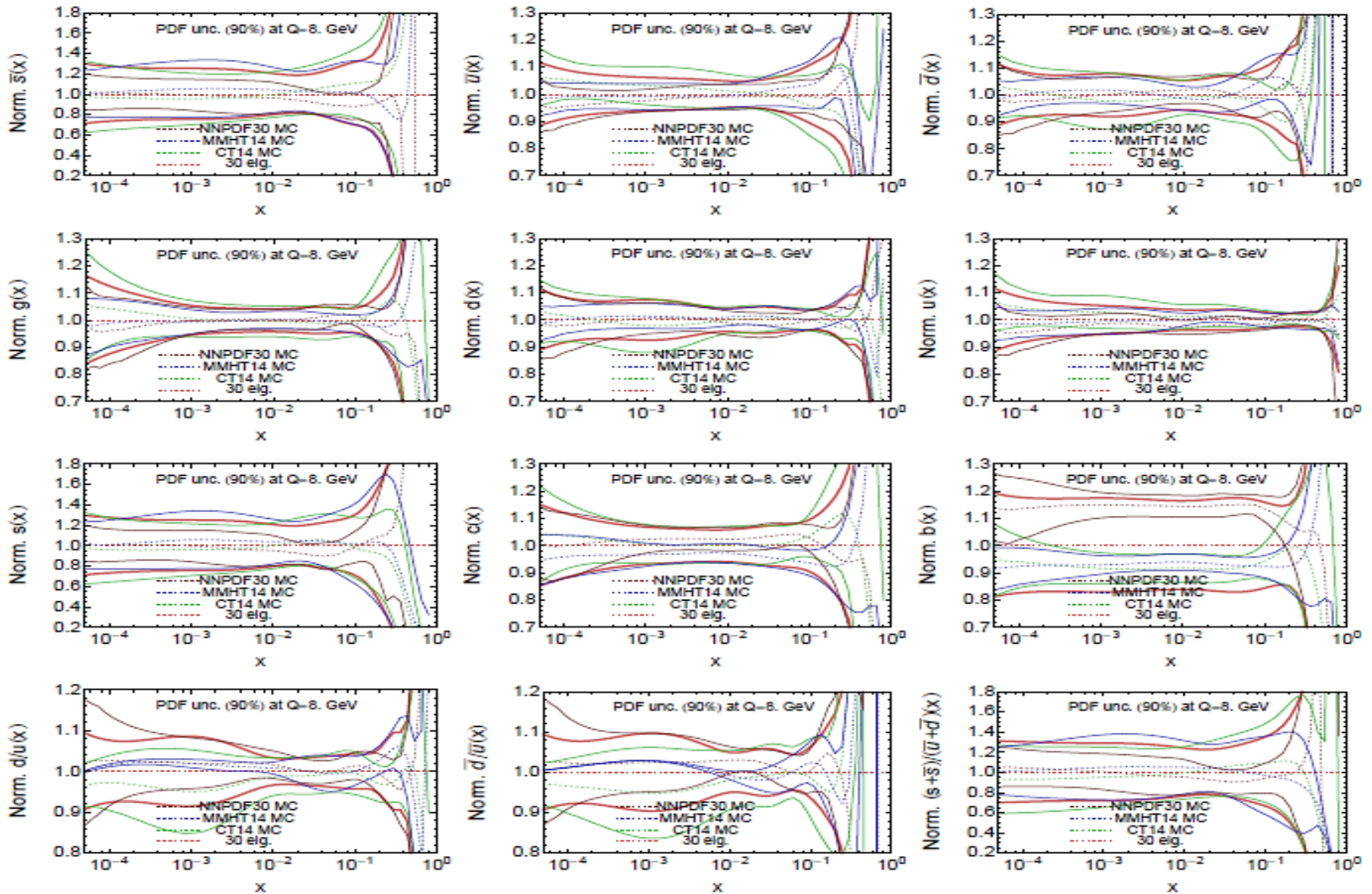
The reason for **forty+** META PDFs

- Crudely, **at least** 20+ PDF parameters (40+ error sets) are needed to reproduce input PDF uncertainties and correlations **in any reduction approach** (7 PDF flavors in >3 independent dynamic regions at small, intermediate, and large x)
- We find that the META2.0 and CMC ensembles with 40 error sets each reproduce key features of 600 replicas with about the same accuracy. The 60-member META ensemble retains even more information, at the price of introducing additional sets.

- **Initial scale $Q_0 = 8 \text{ GeV}$** , selected sufficiently above the bottom mass. Below Q_0 , different heavy-quark schemes must be used in hard cross sections for CT, MMHT, NNPDF PDFs \Rightarrow The user must be made aware they cannot be naively combined.
- At $Q > Q_0$, META PDFs can be used with zero-mass hard cross sections, 5 active flavors. Individual heavy-quark schemes are unnecessary. PDFs can be combined.

Is Gaussian approximation sufficient?

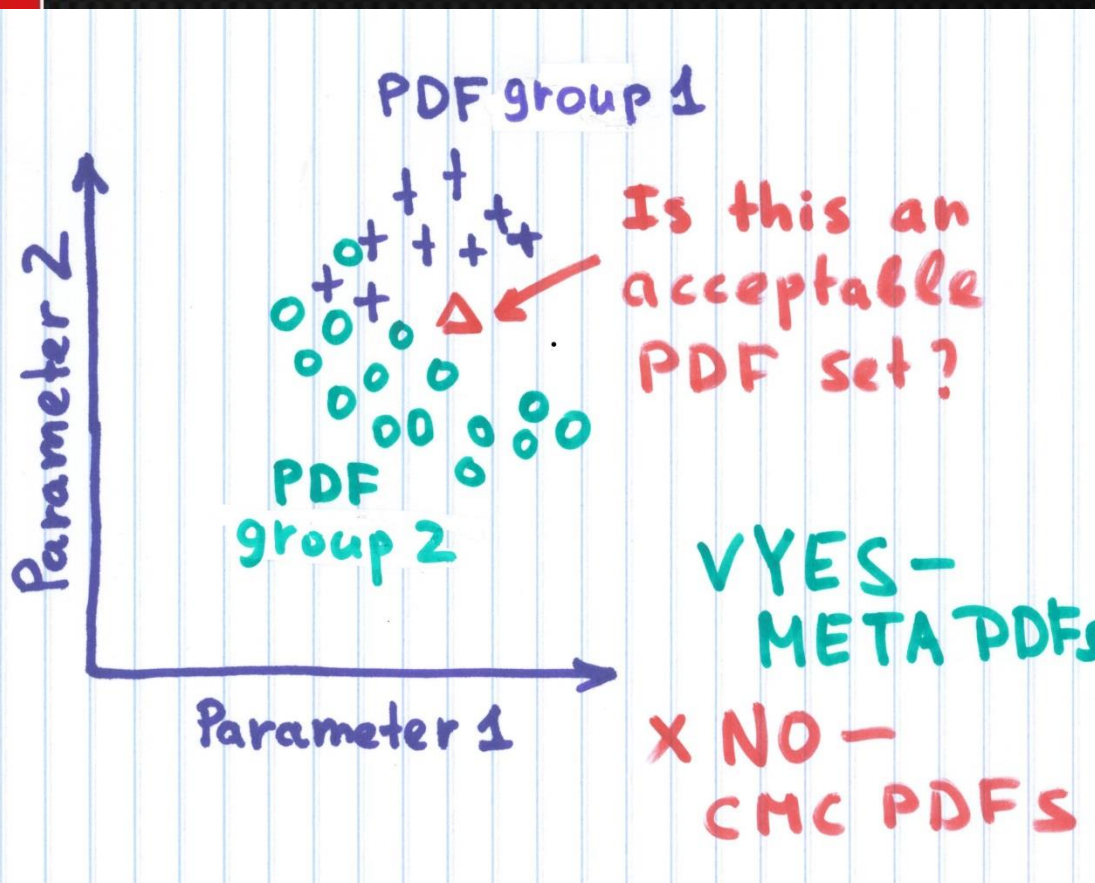
- Non-Gaussian features of given input PDFs can be reproduced in the META approach, e.g., by selecting a parametrization form to convert non-Gaussian probability distribution into a quasi-Gaussian one
- At the moment, this seems impractical:
 - Much of non-Gaussianity is associated with transient features of input PDFs (“noise”); it has large uncertainty of its own, varies between generations of PDF ensembles even from one group
 - Propagation of non-Gaussian uncertainties is contrived; e.g., is the “**central value**” of a non-Gaussian distribution equal to its mean, median, or mode?
 - Non-Gaussian features of truly physical origin, such as positivity of PDFs, are reproduced by the META2.0 method



The META60 ensemble “averages out” non-Gaussian features of input PDFs and their ratios from CT, MMHT, NNPDF MC sets

Gaussian confidence regions are convex

Gaussian error propagation implies that a linear combination of solutions from the input PDF groups is also an allowed solution.



This may be a reasonable assumption in many practical situations, e.g., when the input PDF groups do not sample all possible parameter space.

This feature is not true for Monte-Carlo sampling.

Mathematica module MP4LHC

- Implements all necessary functions to perform META analysis, data set diagonalization, etc. within ≈ 1 day
- **IMPORTANT:** Mathematica finds **all** eigenvalues of the Hessian matrix H_{ij} with high accuracy. Eigenvalues of H_{ij} for a typical PDF set span up to 10 orders of magnitude. Common diagonalization codes can lose precision dramatically. For CTEQ Hessian analysis, Pumplin had to revise CERN MINUIT to evaluate small eigenvalues, prevent wrong solutions for poorly constrained eigenvector sets.
- MP4LHC utilizes versatile *Mathematica* methods for singular value decomposition of H_{ij} . It can achieve essentially arbitrary accuracy for any reasonable number of parameters

Progress in developing the combination procedure

Two methods for combination of PDFs were extensively compared, with promising results:

1. Meta-parametrizations + MC replicas + Hessian data set diagonalization

(J. Gao, J. Huston, P. Nadolsky, 1401.0013)

2. Compression of Monte-Carlo replicas

(Carazza, Latorre, Rojo, Watt, 1504:06469)

Both procedures start by creating a combined ensemble of MC replicas from all input ensembles (G. Watt, R. Thorne, 1205.4024; S. Forte, G. Watt, 1301.6754). They differ at the second step of reducing a large number of input MC replicas (~ 300) to a smaller number for practical applications (13-100 in the META approach; 40 in the CMC approach). The core question is how much input information to retain in the reduced replicas in each Bjorken- x region.

Benchmark comparisons of two combination methods.

Work plan (from Benasque workshop)

Input MC ensemble: **NNPDF3.0+CT14+MMHT14 NNLO**, with **$\alpha_s(M_Z)=0.118$**

Convert to 300 replicas in LHAPDF6 format at $Q_0 = 8$ GeV (above the bottom mass), using two independent codes (JR and JG). Cross-check that results are identical.

Done. The results from two groups agree. Mild differences are due to random variations in the generation of MC replicas.

In each approach, reduce the number of replicas to the minimal number that retains 1% or 5% accuracy in reproducing the following properties of the input ensemble:

- Means, 68% c.l. PDF uncertainties, higher moments and asymmetry (skewness), PDF-PDF correlations.
Done. Ensembles with 40-100 META PDFs and 40 CMC replicas broadly agree.
- Predictions for the standard candle LHC observables used in the META paper: ggHiggs, ttbar, W,Z [Jun]
Done. Broad agreement.
- Differential LHC distributions using NNPDF3.0 applgrids, supplemented with some new aMCfast grids [Juan]

A variety of comparisons collected at <http://bit.ly/1KFoStg>

Benchmark comparisons of CMC and META PDFs

CMC ensembles with 40 replicas and META ensembles with 40-100 replicas are compared with the full ensembles of 300-600 MC replicas.

Accuracy of both combination procedures is already competitive with the 2010 PDF4LHC procedure, can be further fine-tuned by adjusting the final number of replicas.

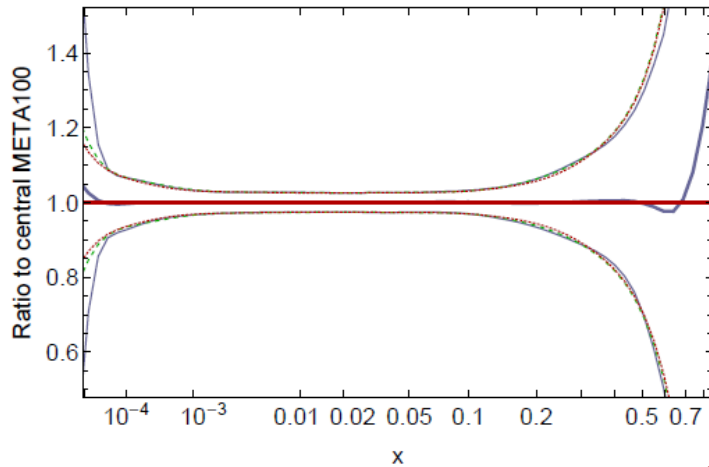
Error bands:

In the (x, Q) regions covered by the data, the agreement of 68%, 95% c.l. intervals is excellent. The definition of the central PDFs and c.l. intervals is ambiguous in extrapolation regions, can differ even within one approach. E.g., differences between mean, median, mode “central values”.

Reduction, META ensemble: 600 \rightarrow 100 \rightarrow 60 error sets

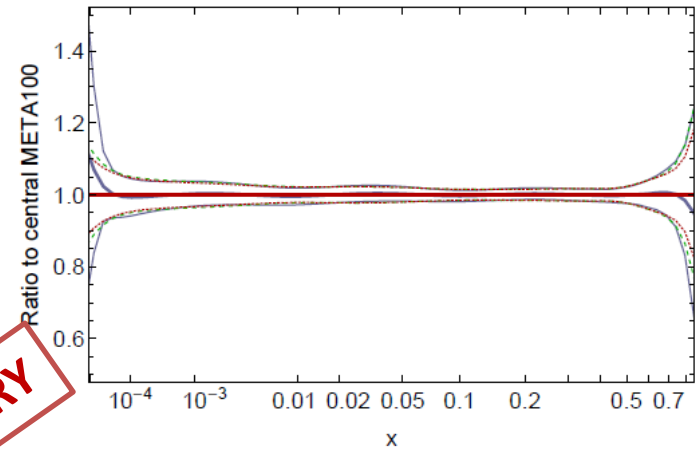
$g(x, Q)$ at $Q=8$ GeV at 68% c.l.

META600 (solid), META100 (dashed), META60 (dotted)



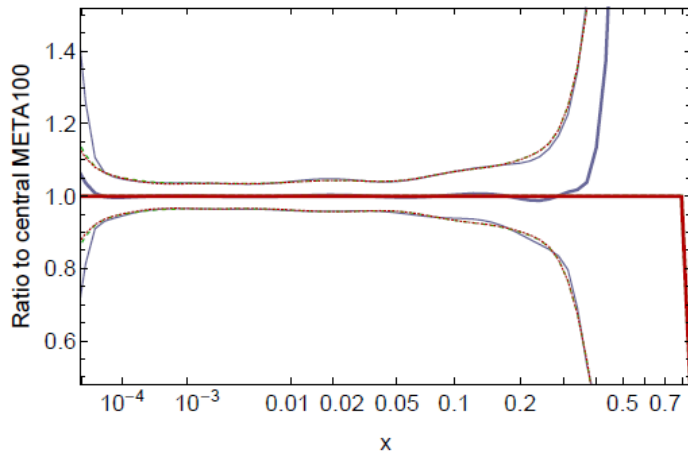
$u(x, Q)$ at $Q=8$ GeV at 68% c.l.

META600 (solid), META100 (dashed), META60 (dotted)



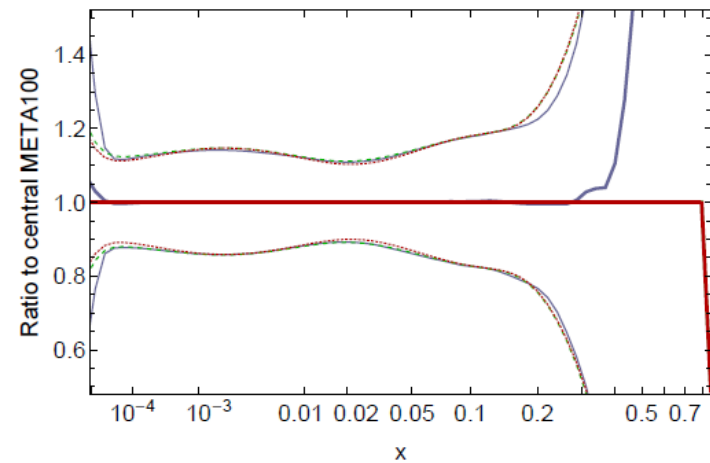
$\bar{d}(x, Q)$ at $Q=8$ GeV at 68% c.l.

META600 (solid), META100 (dashed), META60 (dotted)



$\bar{s}(x, Q)$ at $Q=8$ GeV at 68% c.l.

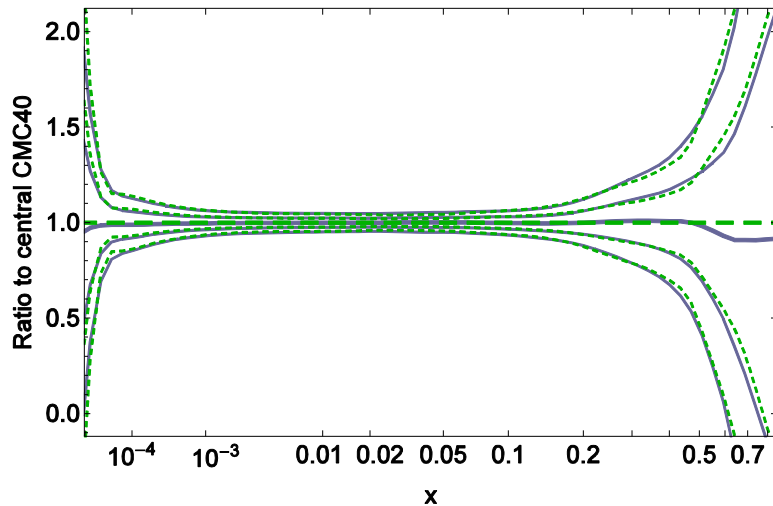
META600 (solid), META100 (dashed), META60 (dotted)



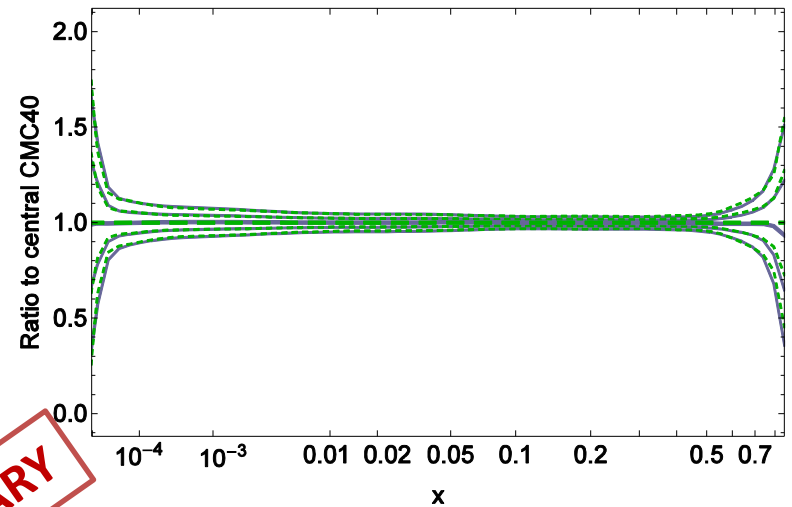
PRELIMINARY

Reduction, CMC ensemble: 300 \rightarrow 40 replicas

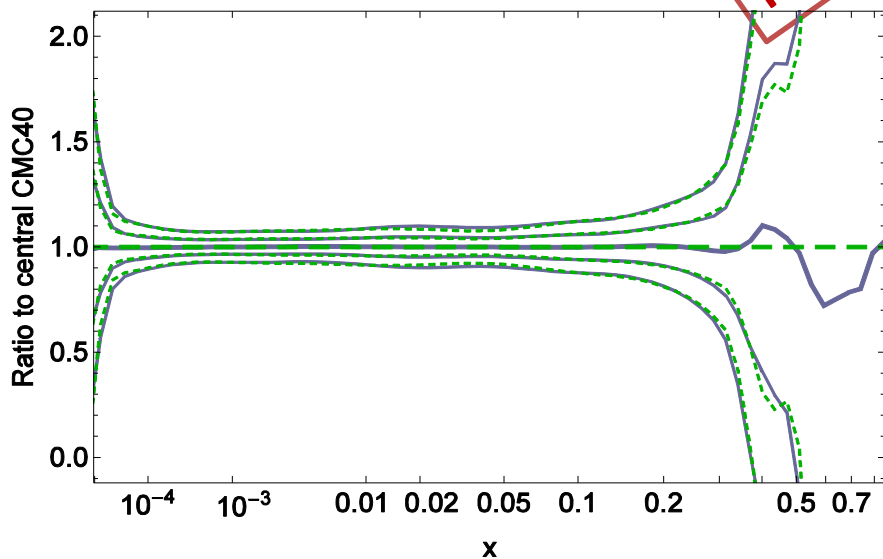
$g(x,Q)$ at $Q=8$ GeV at 1σ and 2σ
CMC40 (dashed), CMC300 (solid)



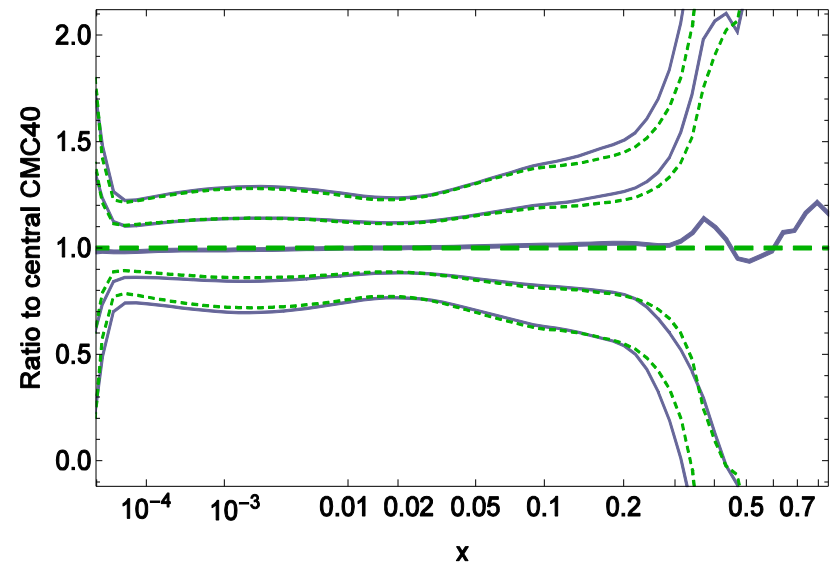
$u(x,Q)$ at $Q=8$ GeV at 1σ and 2σ
CMC40 (dashed), CMC300 (solid)



$d(x,Q)$ at $Q=8$ GeV at 1σ and 2σ
CMC40 (dashed), CMC300 (solid)

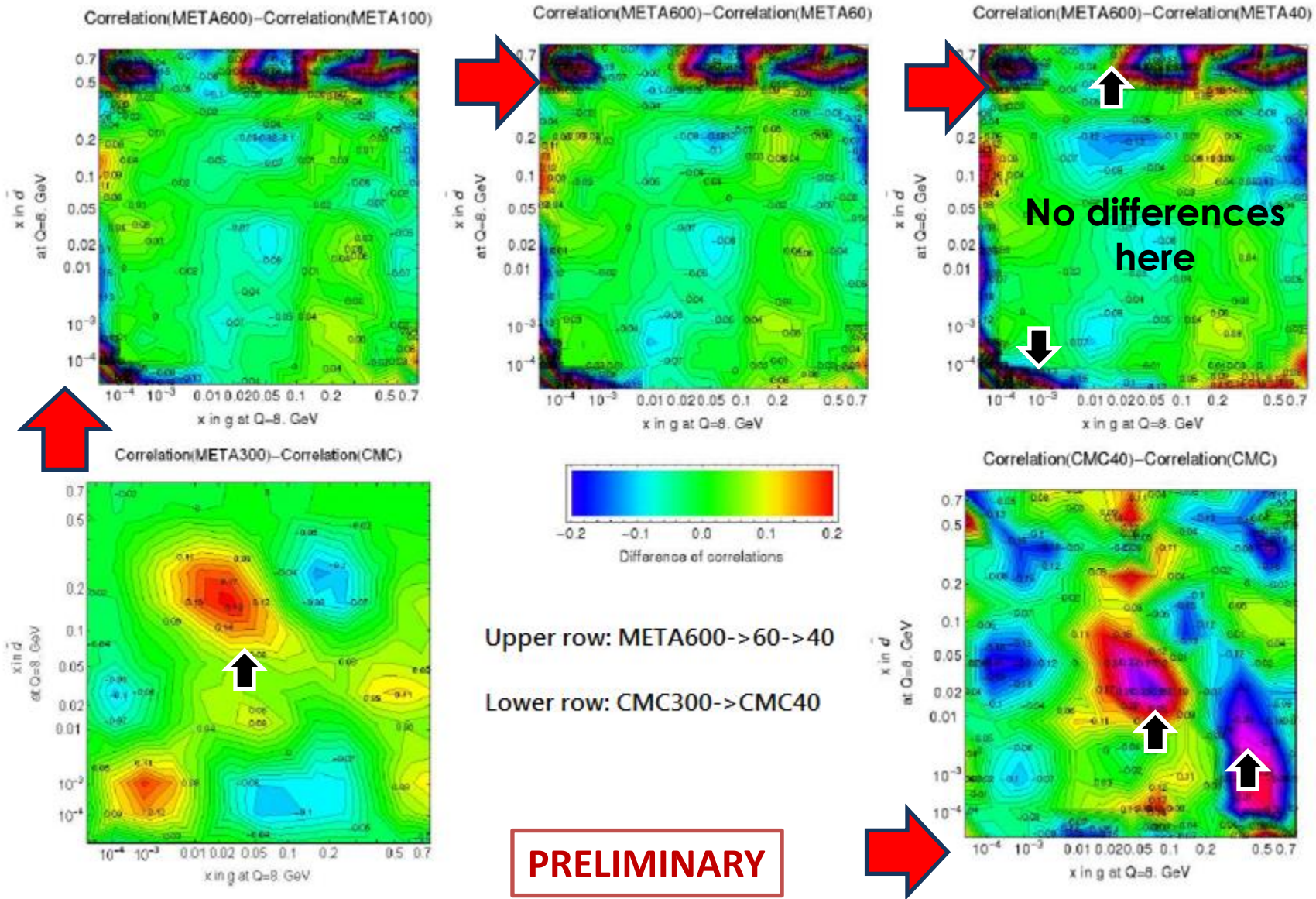


$\bar{s}(x,Q)$ at $Q=8$ GeV at 1σ and 2σ
CMC40 (dashed), CMC300 (solid)



PRELIMINARY

PDF-PDF correlation, example: $\bar{d}(x, Q)$ vs $g(x, Q)$ at $Q = 8 \text{ GeV}$



Benchmark comparisons, general observations II

PDF-PDF correlations:

Correlations of META300 and CMC300 ensembles differ by up to ± 0.2 as a result of fluctuations in replica generation

META40 PDFs faithfully reproduce PDF-PDF correlations of the META600 PDFs in the regions with data; fail to reproduce correlations in extrapolation regions \Rightarrow *next slide, upper row*

CMC40 PDFs better reproduce correlations of CMC300 in extrapolation regions; lose more accuracy in (x, Q) regions with data, but still within acceptable limits \Rightarrow *next slide, lower row*

These patterns of correlations persist at the initial scale

$Q_0 = 8$ GeV as well as at EW scales

To summarize, the meta-parametrizations and Hessian method have been thoroughly validated

- A general and intuitive framework. Implemented in a public Mathematica module MP4LHC
- The PDF parameter space of all input ensembles is visualized explicitly.
- Data combination procedures familiar from PDG can be applied to each meta-PDF parameter
- Asymmetric Hessian errors can be computed, similar to CT14 approach
- Effective in data reduction; makes use of diagonalization of the Hessian matrix in the Gaussian approximation. Reproduces correlations between Higgs signals and backgrounds with just 13 META –LHCH PDFs.
- **Work plan: prepare META2.0 PDFs and MP4LHC for release during the Les Houches week**

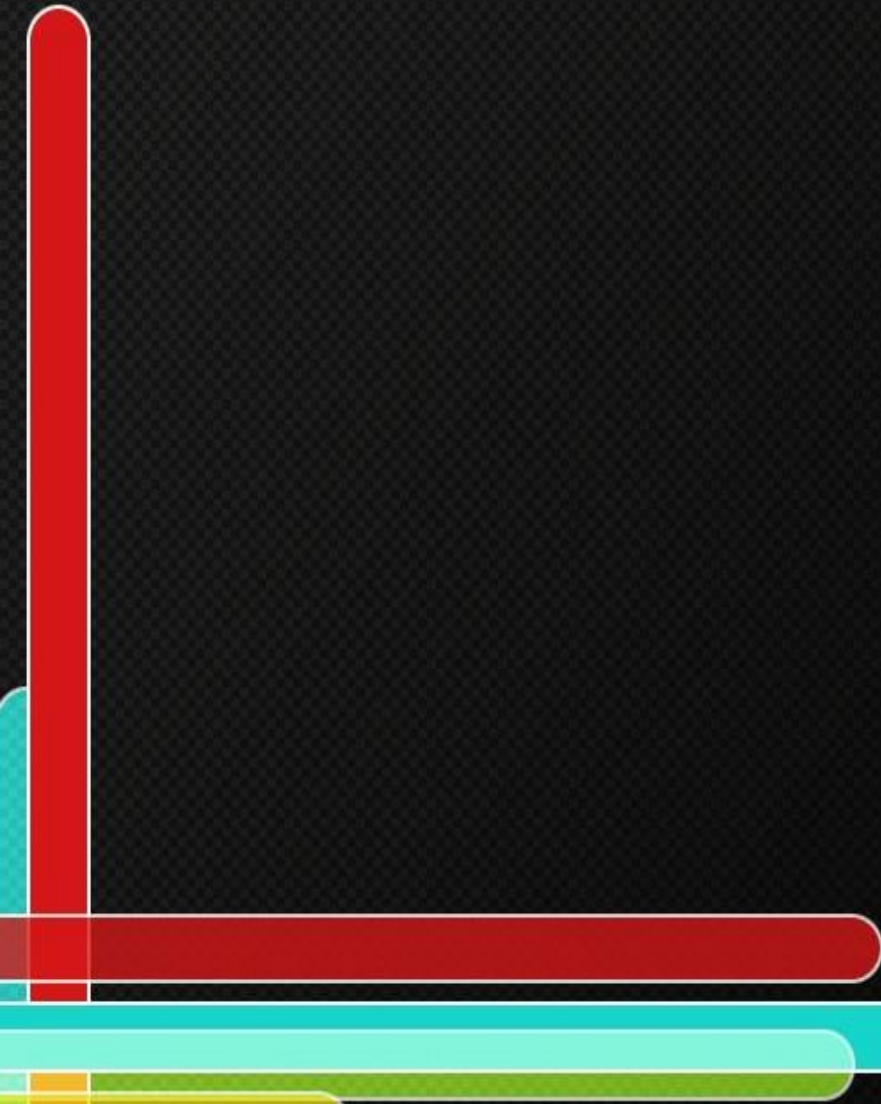
Implications for the PDF4LHC prescription

- In our opinion, MP4LHC and MC2Hessian, and developments in these approaches, both realize variations of a generic meta-parametrization method.
- Meta-analysis could be stated to be the default framework for the PDF4LHC prescription
- The two methods will provide specific realizations for the generic method and allow for a variety of quality cross checks

Back-up slides



META 2.0 PDFs



Meta-parameters of 5 sets and META PDFs

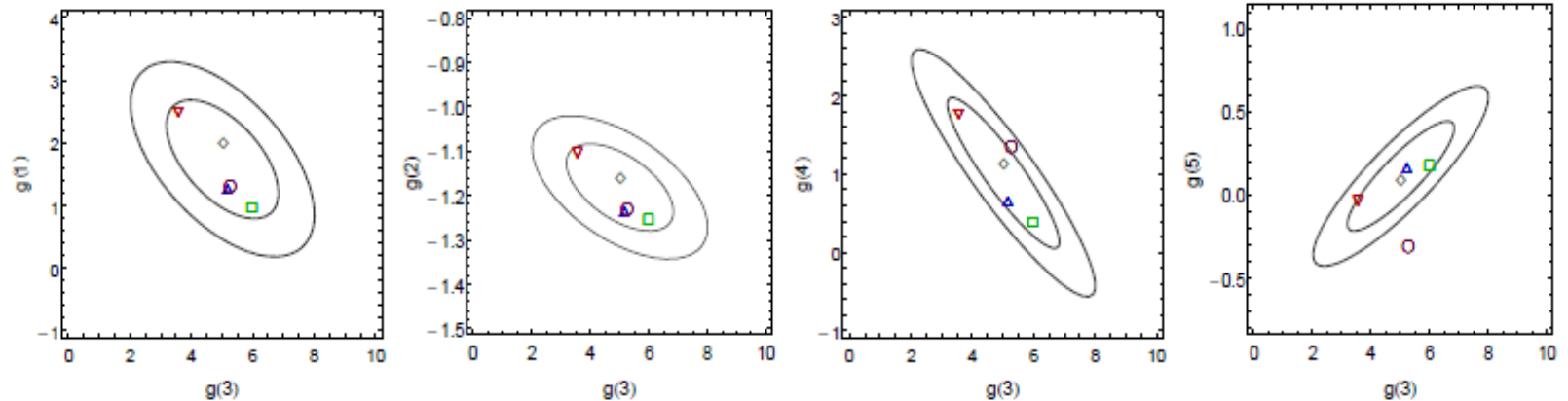


Figure 16: Comparison of META PDF confidence intervals with central NNLO PDFs of the input PDF ensembles in space of meta-parameters a_{1-5} for the gluon PDF. Up triangle, down triangle, square, diamond, and circle correspond to the best-fit PDFs from CT10, MSTW, NNPDF, HERAPDF, and ABM respectively. The ellipses correspond to 68 and 90% c.l. ellipses of META PDFs.

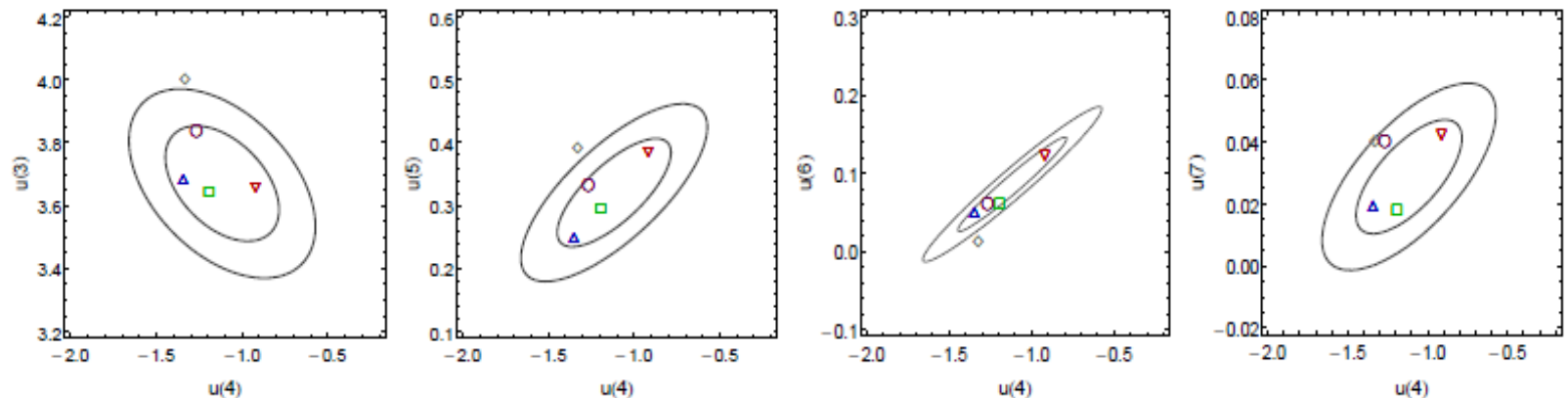


Figure 17: Same as Fig. 16, for a_{3-7} of the u quark PDF.

Merging PDF ensembles

The ensembles can be merged by averaging their meta-parameters. For CT10, MSTW, NNPDF ensembles, unweighted averaging is reasonable, given their similarities.

For any parameter a_i , ensemble g with N_{rep} initial replicas:

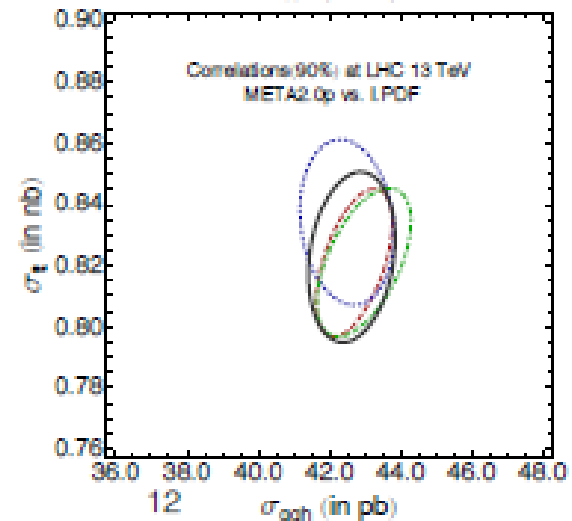
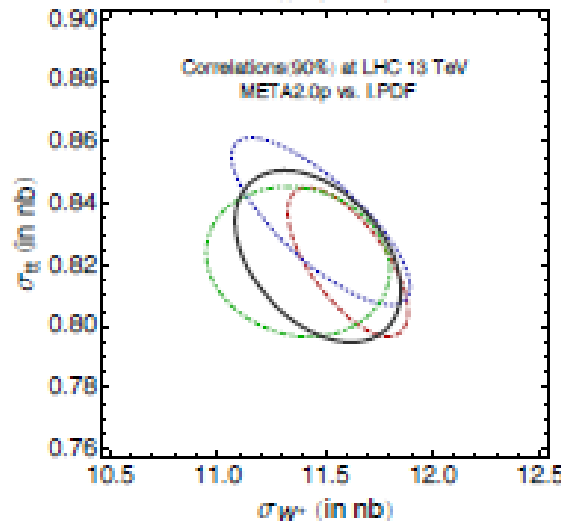
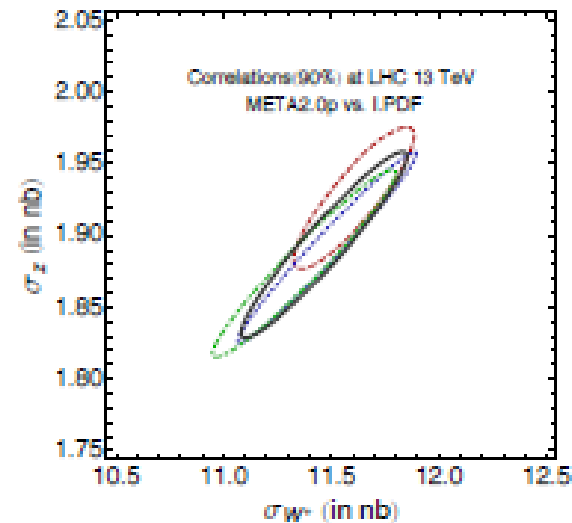
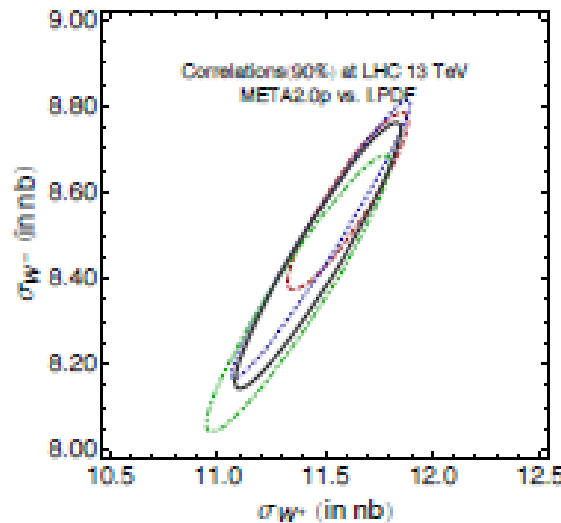
$$\langle a_i \rangle_g = \frac{1}{N_{rep}} \sum_{k=1}^{N_{rep}} a_i(k), \quad \leftarrow \text{Central value on } g$$

$$\text{cov}(a_i, a_j)_g = \frac{N_{rep}}{N_{rep} - 1} \langle (a_i - \langle a_i \rangle_g) \cdot (a_j - \langle a_j \rangle_g) \rangle_g,$$

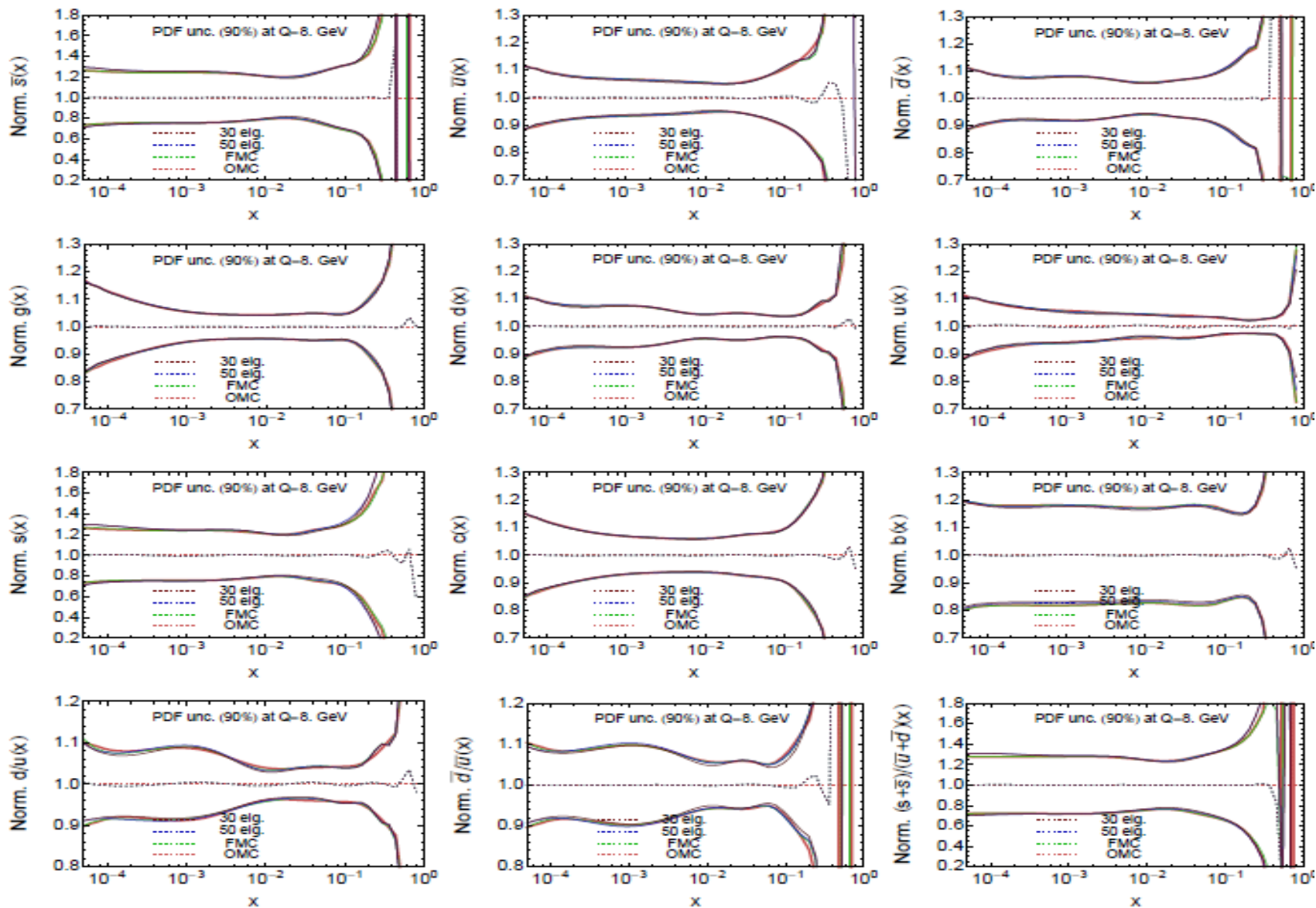
$$(\delta a_i)_g = \sqrt{\text{cov}(a_i, a_i)_g}. \quad \leftarrow \text{Standard deviation on } g$$

META2.0 predictions for LHC observables

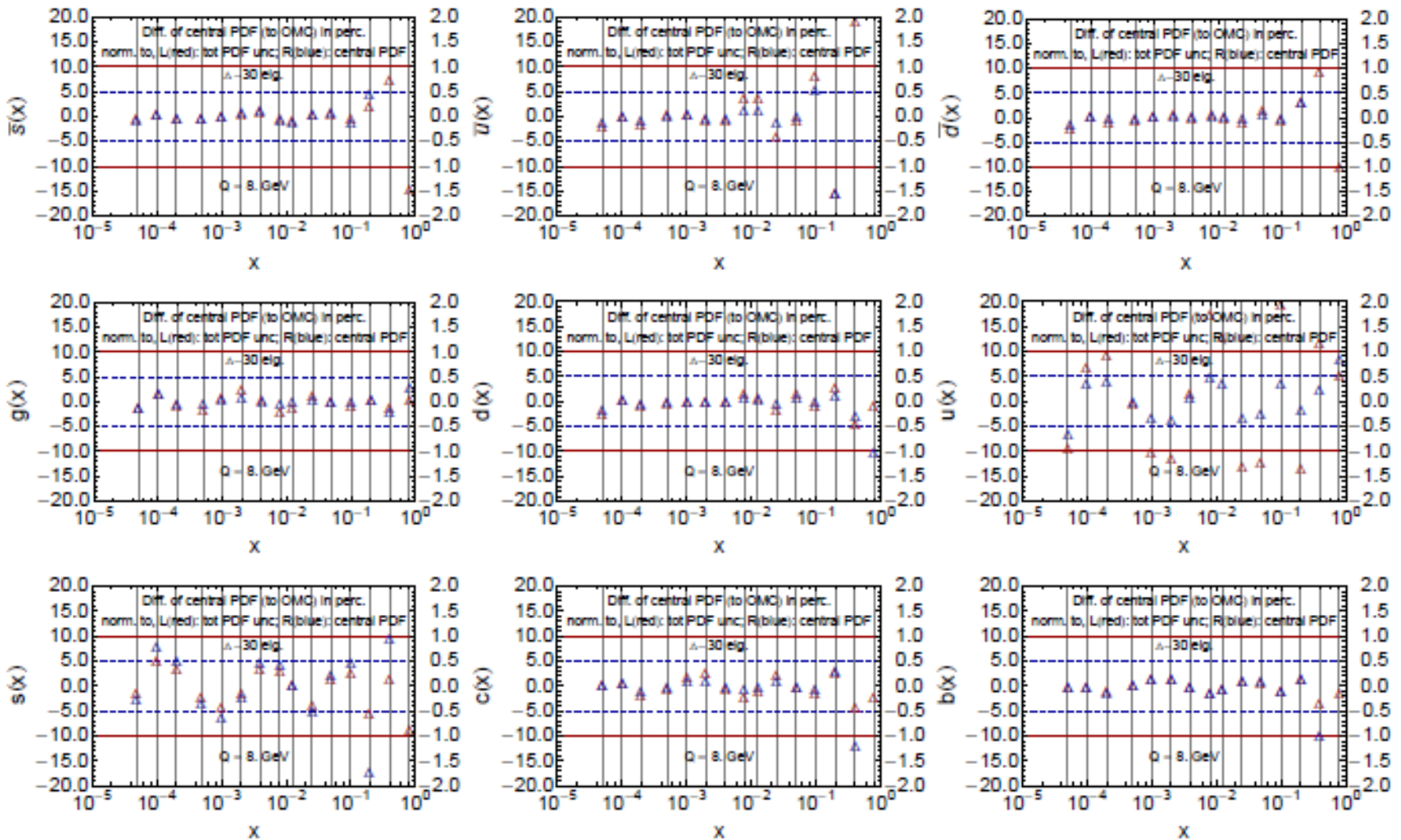
- Currently only have results for META NNLO v2.0p, will add later for v2.1, inclusive observables at 13 TeV



- Blue, CT14p, red, MMHT14, green, NNPDF3.0, black, META2.0p, error ellipse at 90% cl; using Vrap0.9, iHixs1.3, and top++2.0



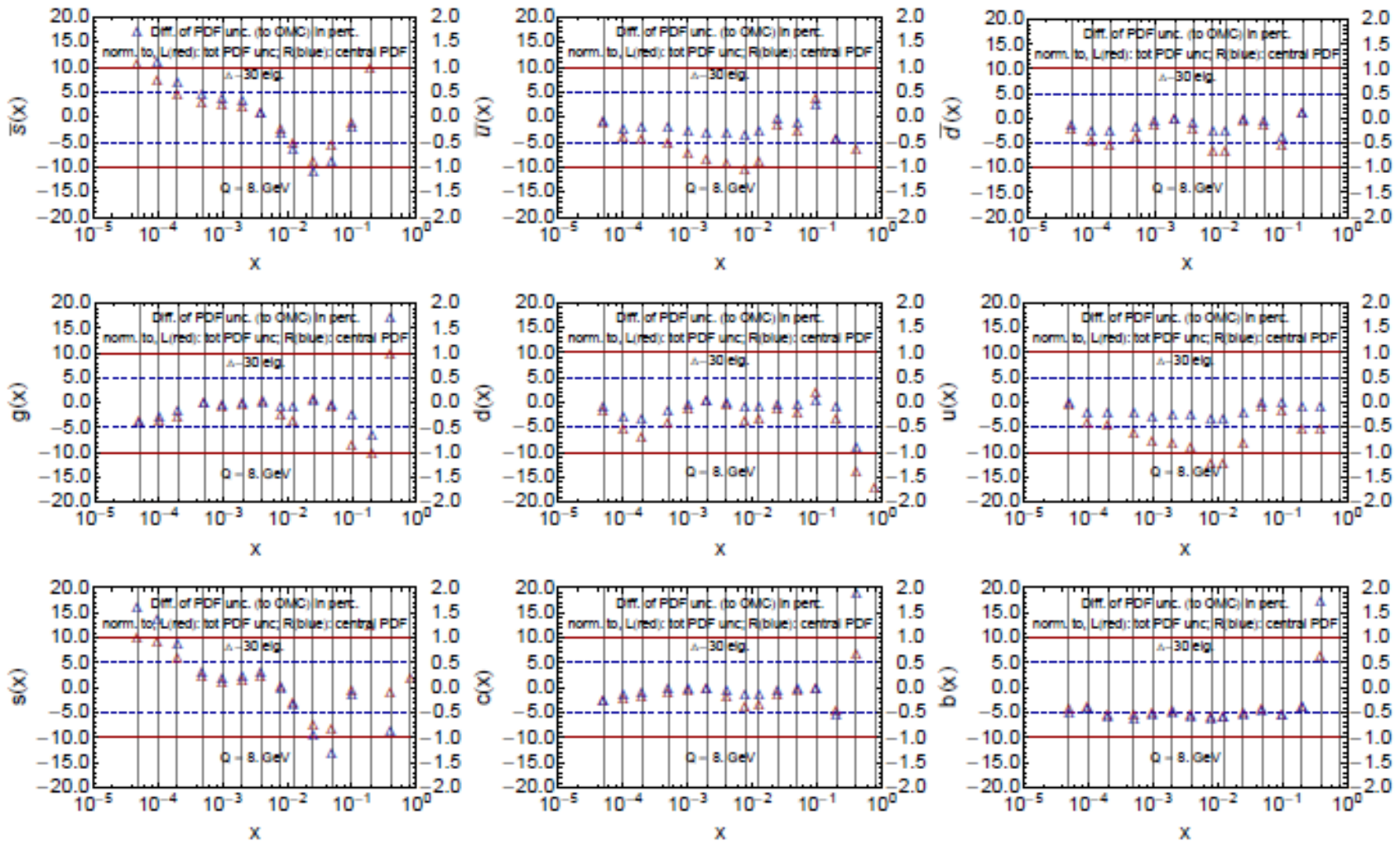
PDF uncertainty bands from original 600 MC replicas (OMC), fitted MC replicas (FMC), META60 and META100. **NO FITTING BIAS OBSERVED!**



Differences in central PDFs between META60 and MC600.

Left axis, red triangles: as percentages of PDF uncertainty

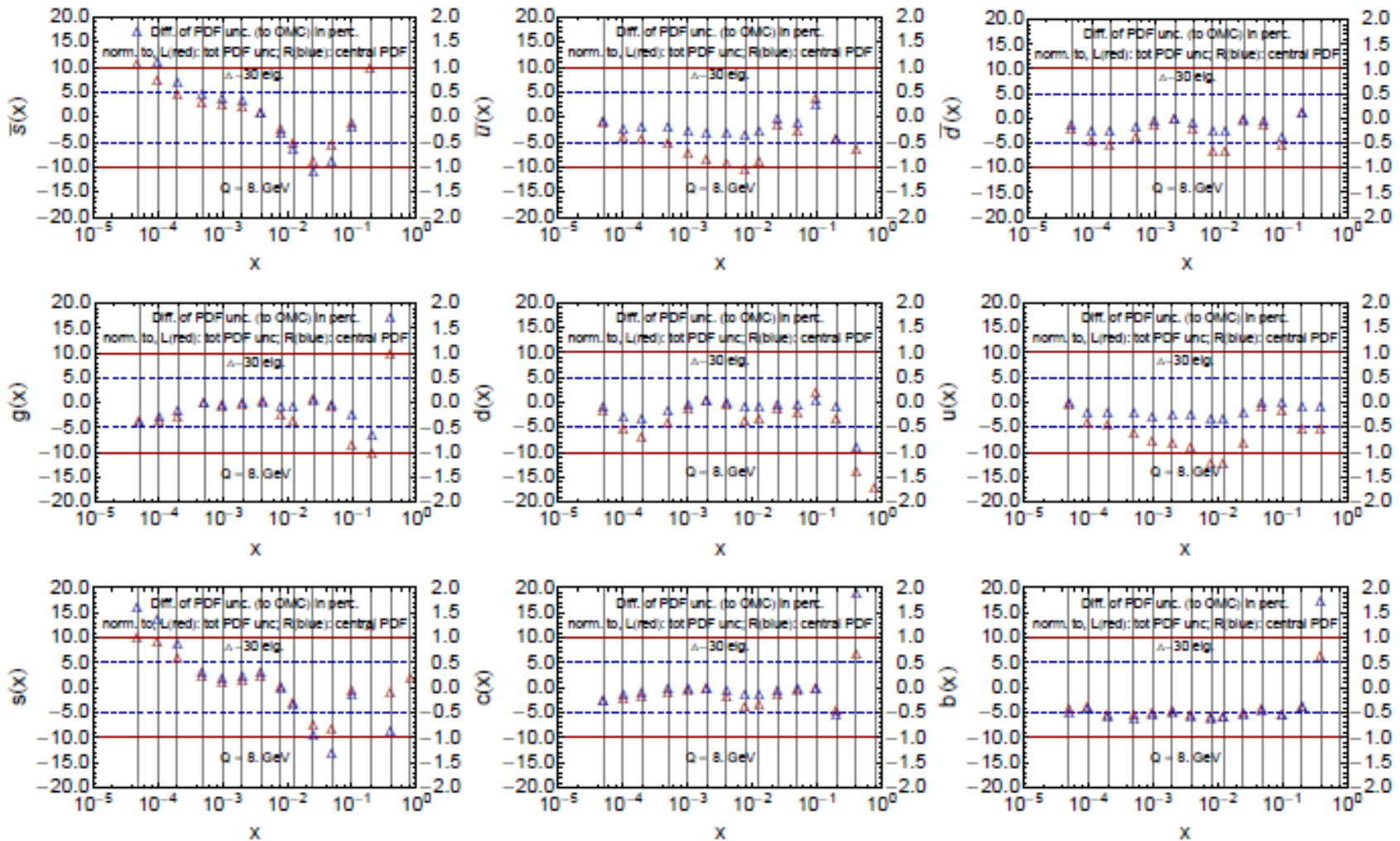
Right axis, blue triangles: as percentages of the central PDF



Differences of PDF uncertainties between META60 and MC600.

Left axis, red triangles: as percentages of the PDF uncertainty

Right axis, blue triangles: as percentages of the central PDF



Differences of PDF uncertainties between META60 and MC600.

Left axis, red triangles: as percentages of the PDF uncertainty

Right axis, blue triangles: as percentages of the central PDF

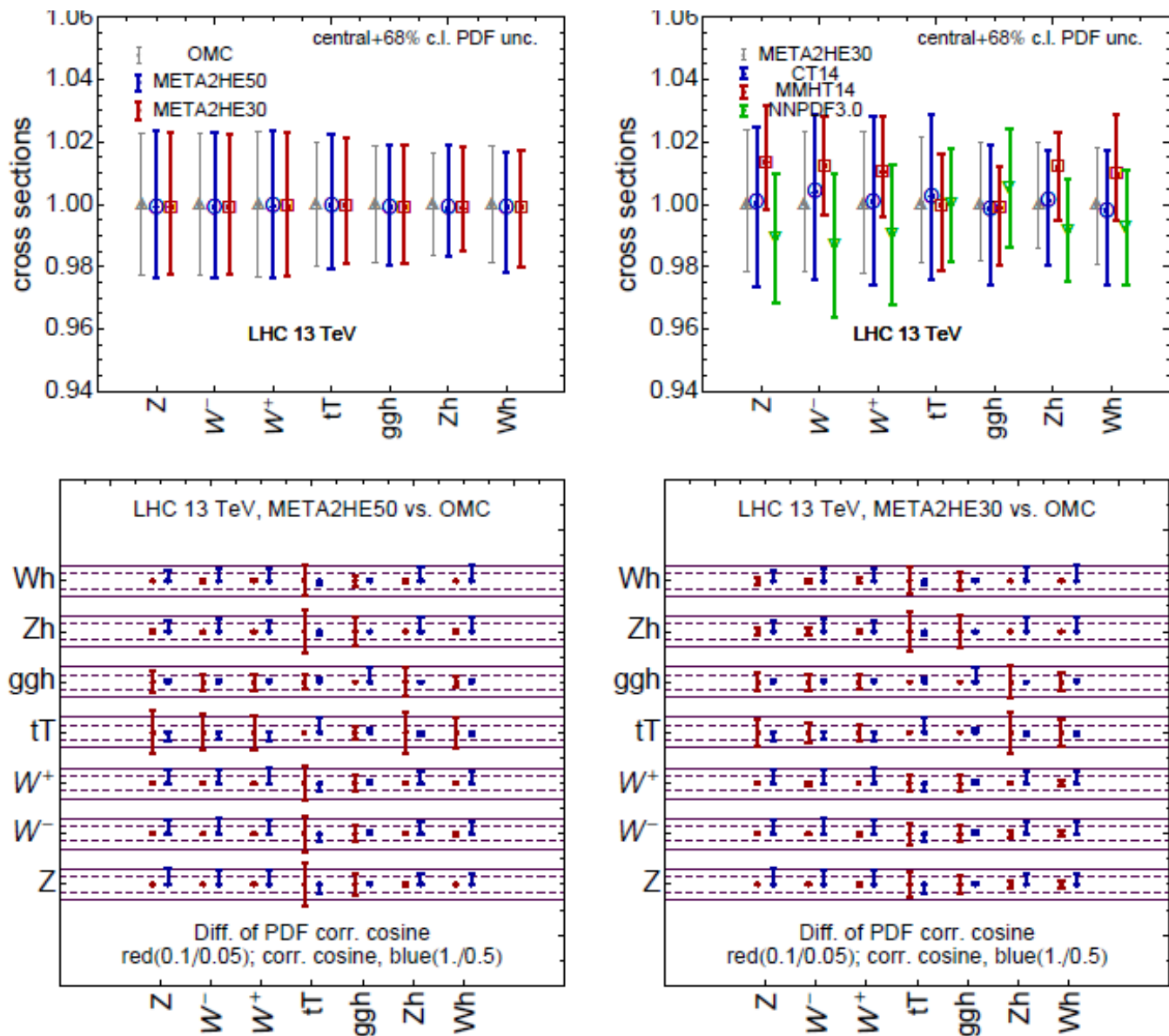
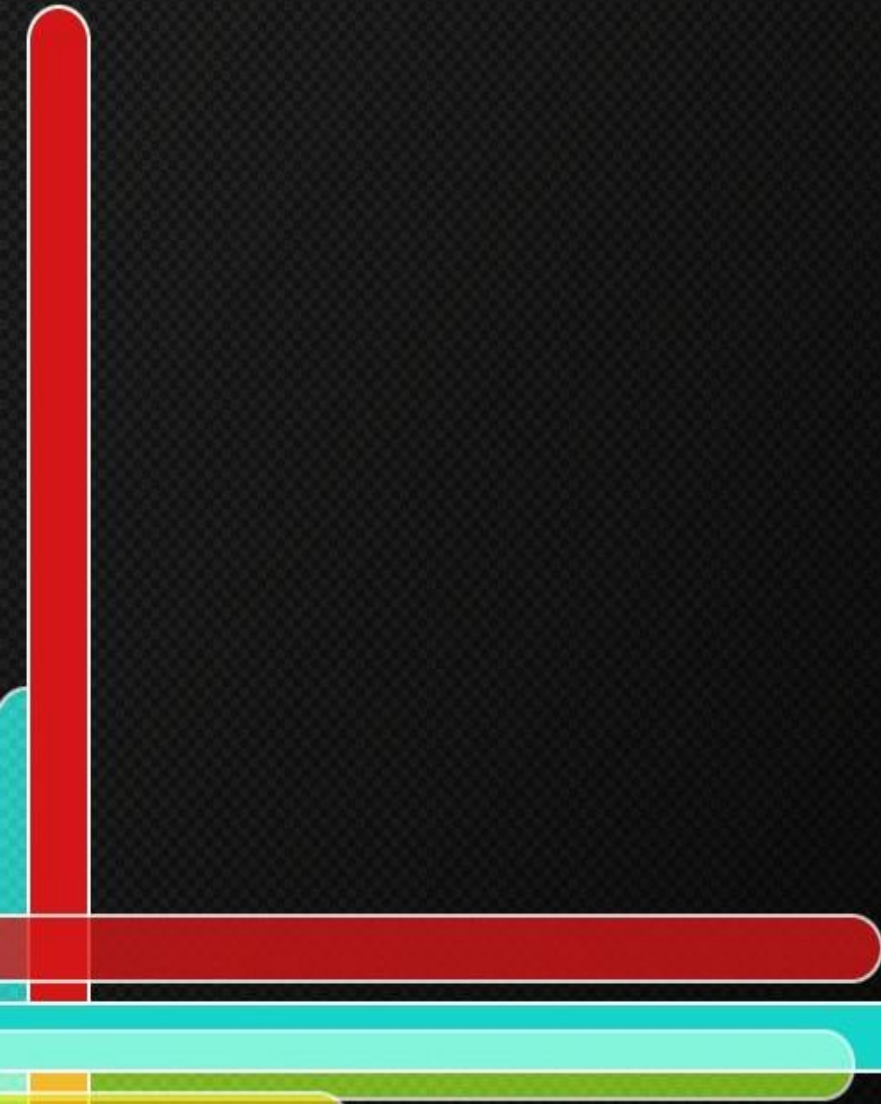


FIG. 9: Comparison on inclusive cross sections for META2HE vs. OMC, and individual PDFs vs. META2HE30; their correlations for META2HE vs. OMC.

Data set diagonalization for Higgs observables



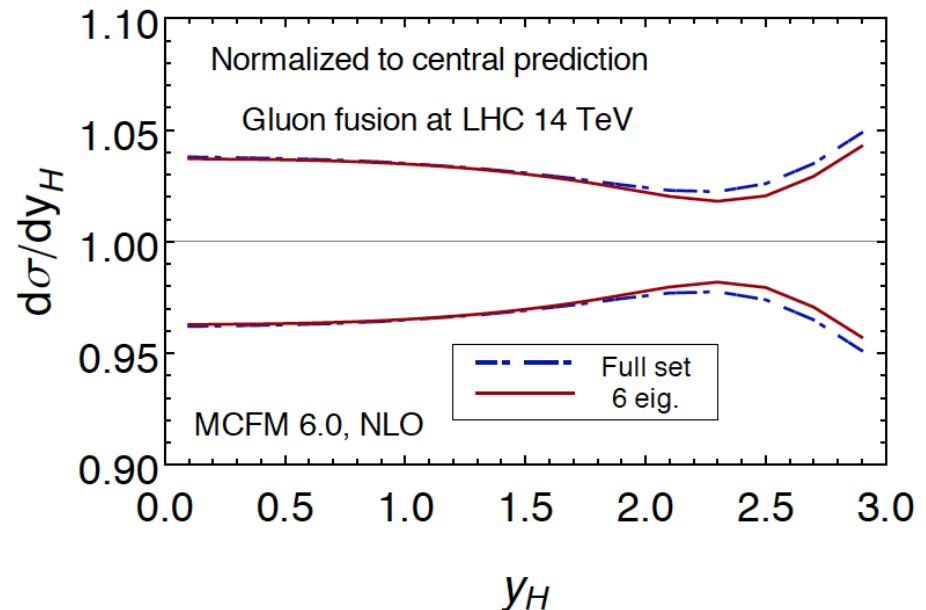
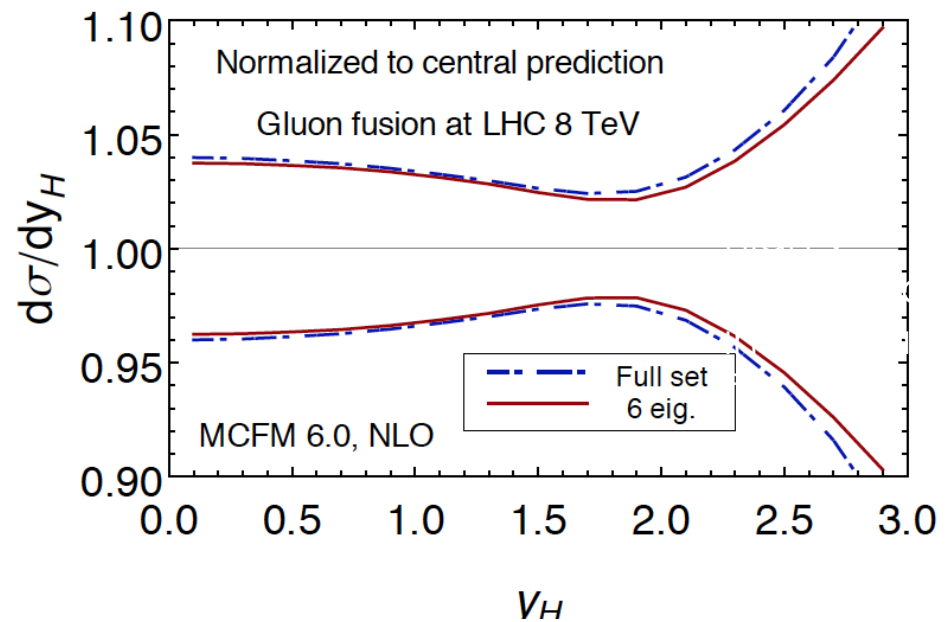
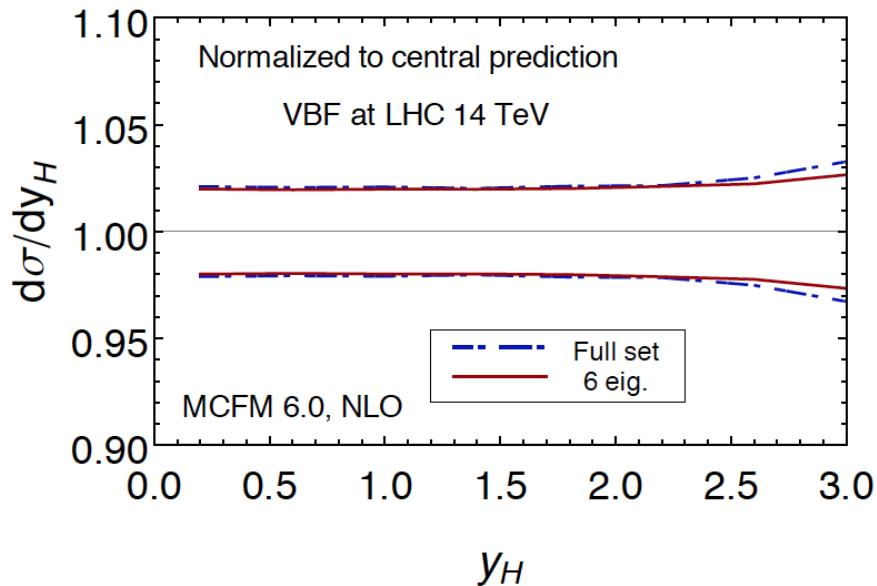
Reduced META ensemble

- Already the general-purpose ensemble reduced the number of error PDFs needed to describe the LHC physics; but we can further perform a data set diagonalization to pick out eigenvector directions important for Higgs physics or another class of LHC processes
- Select global set of Higgs cross sections at 8 and 14 TeV (46 observables in total; more can be easily added if there is motivation)

| production channel | $\sigma(inc.)$ | $\sigma(y_H > 1)$ | $\sigma(p_{T,H} > m_H)$ | scales |
|--------------------------|------------------------|-----------------------|-------------------------|--------------|
| $gg \rightarrow H$ | iHixs1.3 [32] at NNLO | MCFM6.3 [33] at LO | — | m_H |
| $b\bar{b} \rightarrow H$ | iHixs at NNLO | — | — | m_H |
| VBF | VBFNLO2.6 [34] at NLO | same | same | m_W |
| HZ | VHNNLO1.2 [35] at NNLO | CompHEP4.5 [36] at LO | CompHEP at LO | $m_Z + m_H$ |
| HW^\pm | VHNNLO at NNLO | — | — | $m_W + m_H$ |
| HW^+ | CompHEP at LO | same | same | $m_W + m_H$ |
| HW^- | CompHEP at LO | same | same | $m_W + m_H$ |
| $H + 1jet$ | MCFM at LO | same | same | m_H |
| $Ht\bar{t}$ | MCFM at LO | CompHEP at LO | CompHEP at LO | $2m_t + m_H$ |
| HH | Hpair [37] at NLO | — | — | $2m_H$ |

Higgs eigenvector set

- The reduced META eigenvector set does a good job of describing the uncertainties of the full set for typical processes such as ggF or VBF
- But actually does a good job in reproducing PDF-induced correlations and describing those LHC physics processes in which g , \bar{u} , \bar{d} drive the PDF uncertainty (see next slide)



| process | $\sigma_{cen.}$ | δ_{Full} | $\delta_{Diag.}$ | $\sigma_{0.116}^{\alpha_s}$ | $\sigma_{0.12}^{\alpha_s}$ |
|---------------------------|-----------------|------------------|------------------|-----------------------------|----------------------------|
| $gg \rightarrow H$ [pb] | 18.77 | +0.48 -0.46 | +0.48 -0.44 | 18.11 | 19.44 |
| | 43.12 | +1.13 -1.07 | +1.13 -1.04 | 41.68 | 44.64 |
| VBF [fb] | 302.5 | +7.8 -6.7 | +7.6 -6.7 | 303.1 | 301.4 |
| | 878.2 | +19.7 -17.9 | +19.2 -17.3 | 877.3 | 878.2 |
| HZ [fb] | 396.3 | +8.4 -7.3 | +8.1 -7.4 | 393.0 | 399.1 |
| | 814.3 | +14.8 -13.2 | +13.8 -13.0 | 806.5 | 823.3 |
| HW^\pm [fb] | 703.0 | +14.4 -14.4 | +14.3 -14.1 | 697.4 | 708.9 |
| | 1381 | +28 -22 | +26 -22 | 1368 | 1398 |
| HH [fb] | 7.81 | +0.33 -0.30 | +0.33 -0.30 | 7.50 | 8.10 |
| | 27.35 | +0.78 -0.72 | +0.78 -0.68 | 26.48 | 28.24 |
| $t\bar{t}$ [pb] | 248.4 | +9.1 -8.2 | +9.2 -8.1 | 237.1 | 259.1 |
| | 816.9 | +21.4 -19.6 | +21.4 -18.4 | 785.5 | 848.2 |
| $Z/\gamma^*(l^+l^-)$ [nb] | 1.129 | +0.025 -0.023 | +0.024 -0.023 | 1.113 | 1.141 |
| | 1.925 | +0.043 -0.041 | +0.040 -0.037 | 1.897 | 1.951 |
| $W^+(l^+\nu)$ [nb] | 7.13 | +0.14 -0.14 | +0.14 -0.13 | 7.03 | 7.25 |
| | 11.64 | +0.24 -0.23 | +0.22 -0.21 | 11.46 | 11.84 |
| $W^-(l^-\bar{\nu})$ [nb] | 4.99 | +0.12 -0.12 | +0.12 -0.11 | 4.92 | 5.08 |
| | 8.59 | +0.21 -0.20 | +0.19 -0.18 | 8.46 | 8.74 |
| W^+W^- [pb] | 4.14 | +0.08 -0.08 | +0.08 -0.07 | 4.04 | 4.20 |
| | 7.54 | +0.15 -0.14 | +0.14 -0.12 | 7.39 | 7.57 |
| ZZ [pb] | 0.703 | +0.016 -0.014 | +0.015 -0.014 | 0.695 | 0.711 |
| | 1.261 | +0.026 -0.024 | +0.024 -0.022 | 1.256 | 1.271 |
| W^+Z [pb] | 1.045 | +0.019 -0.018 | +0.019 -0.017 | 1.039 | 1.061 |
| | 1.871 | +0.033 -0.031 | +0.029 -0.027 | 1.850 | 1.891 |
| W^-Z [pb] | 0.788 | +0.020 -0.019 | +0.019 -0.018 | 0.780 | 0.791 |
| | 1.522 | +0.034 -0.032 | +0.033 -0.031 | 1.509 | 1.541 |

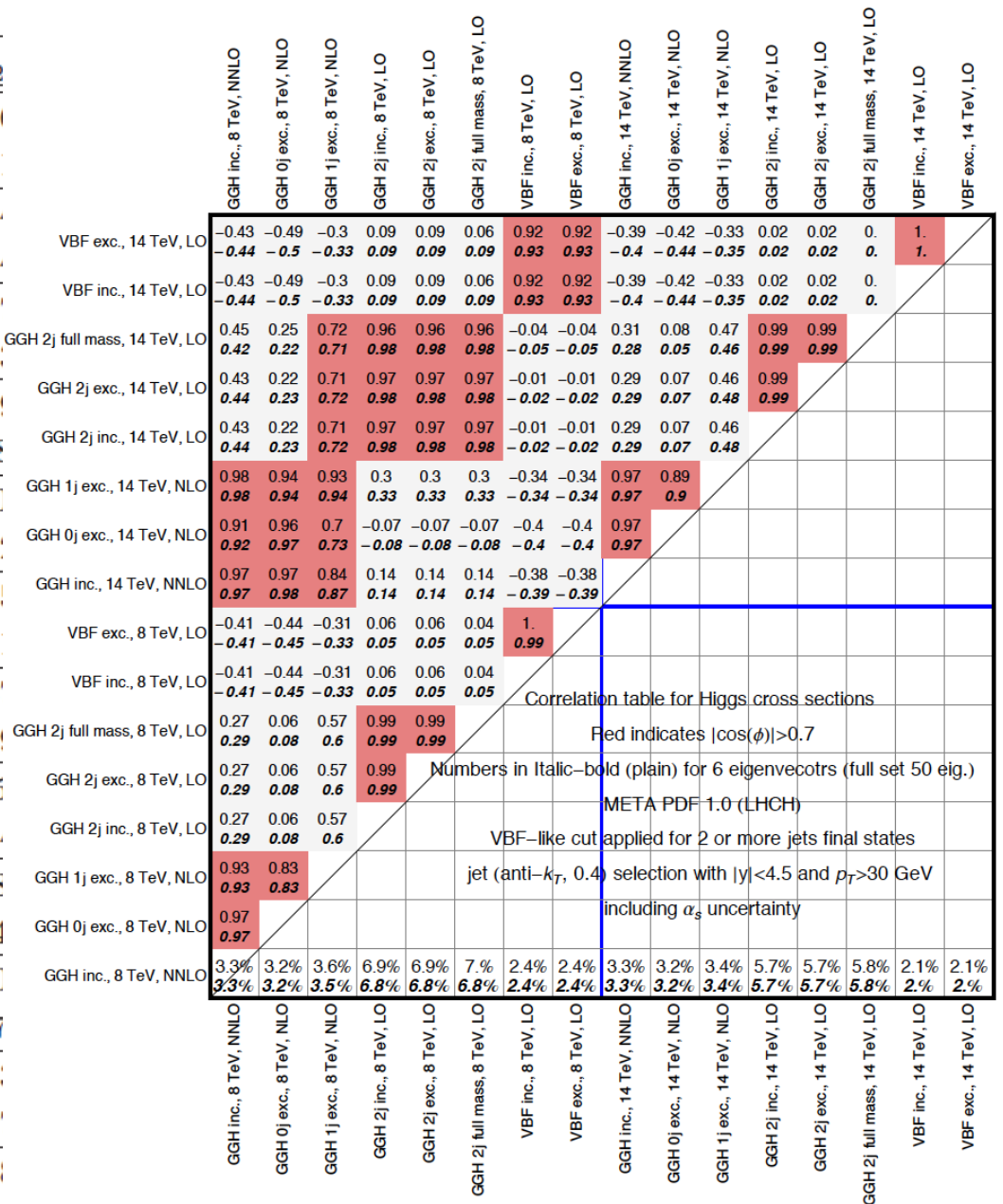
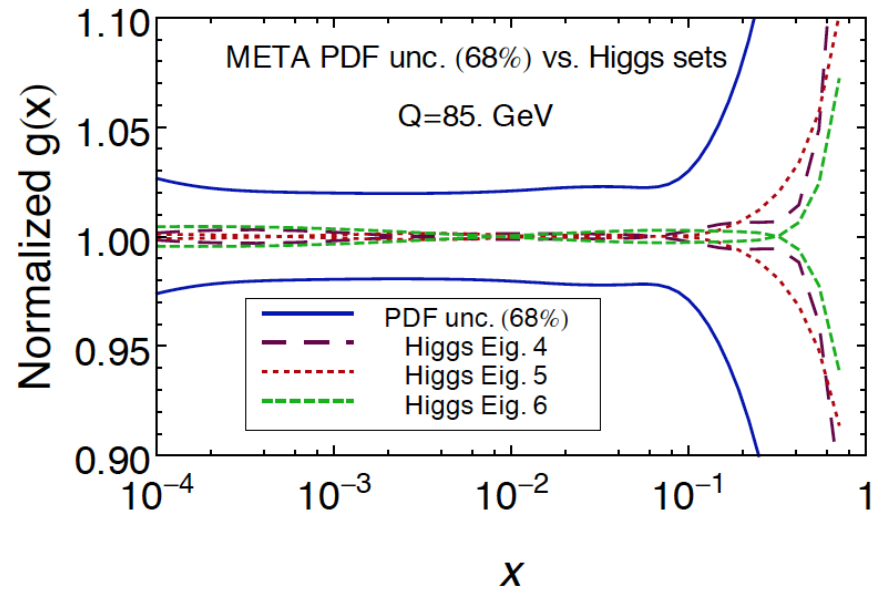
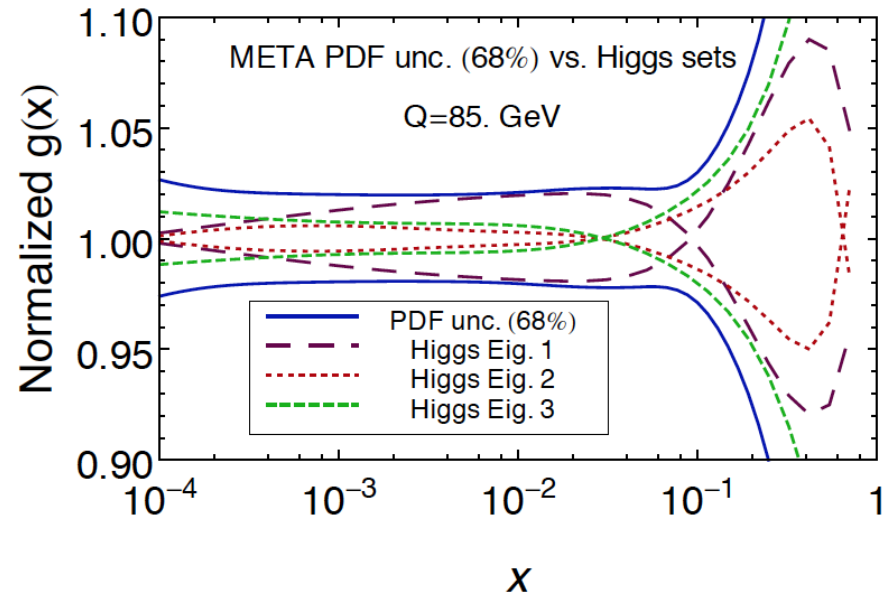


FIG. 7: Same as Fig. 5, with α_s uncertainties included by adding in quadrature.

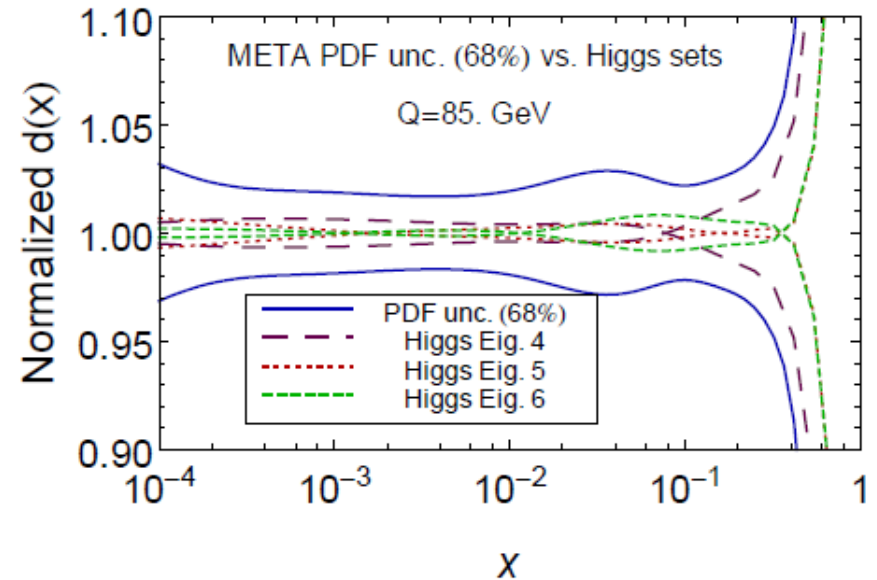
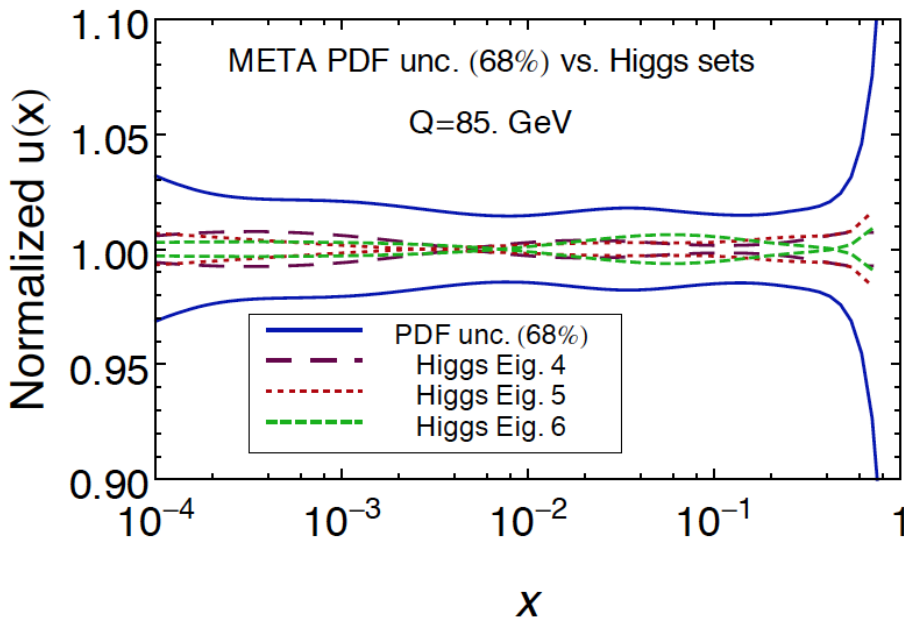
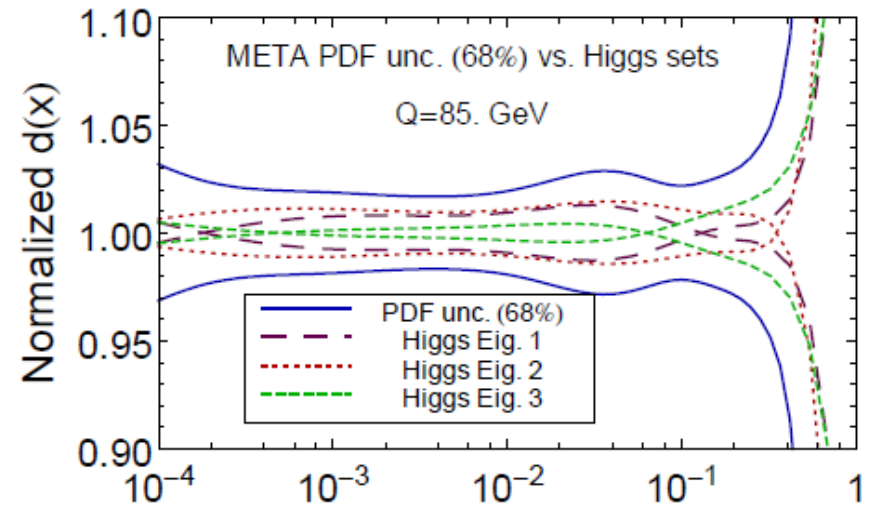
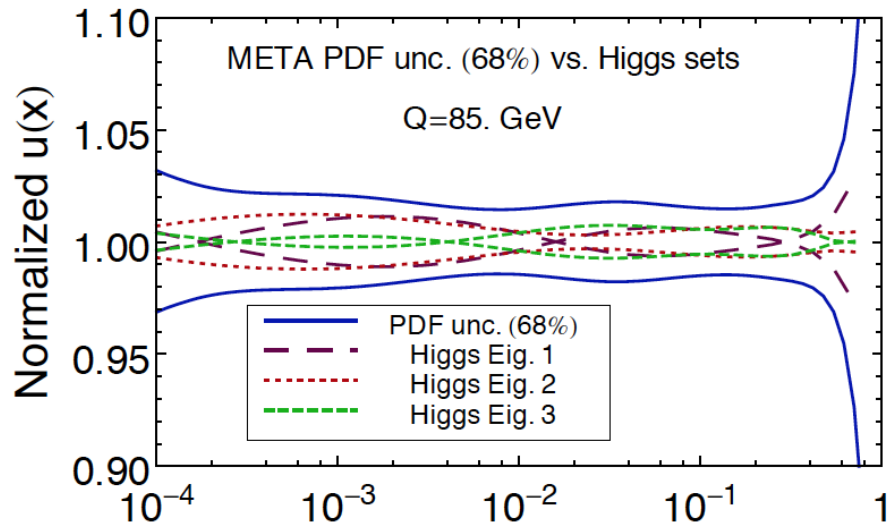
Re-diagonalized eigenvectors...

...are associated with the parameter combinations that drive the PDF uncertainty in Higgs, W/Z production at the LHC

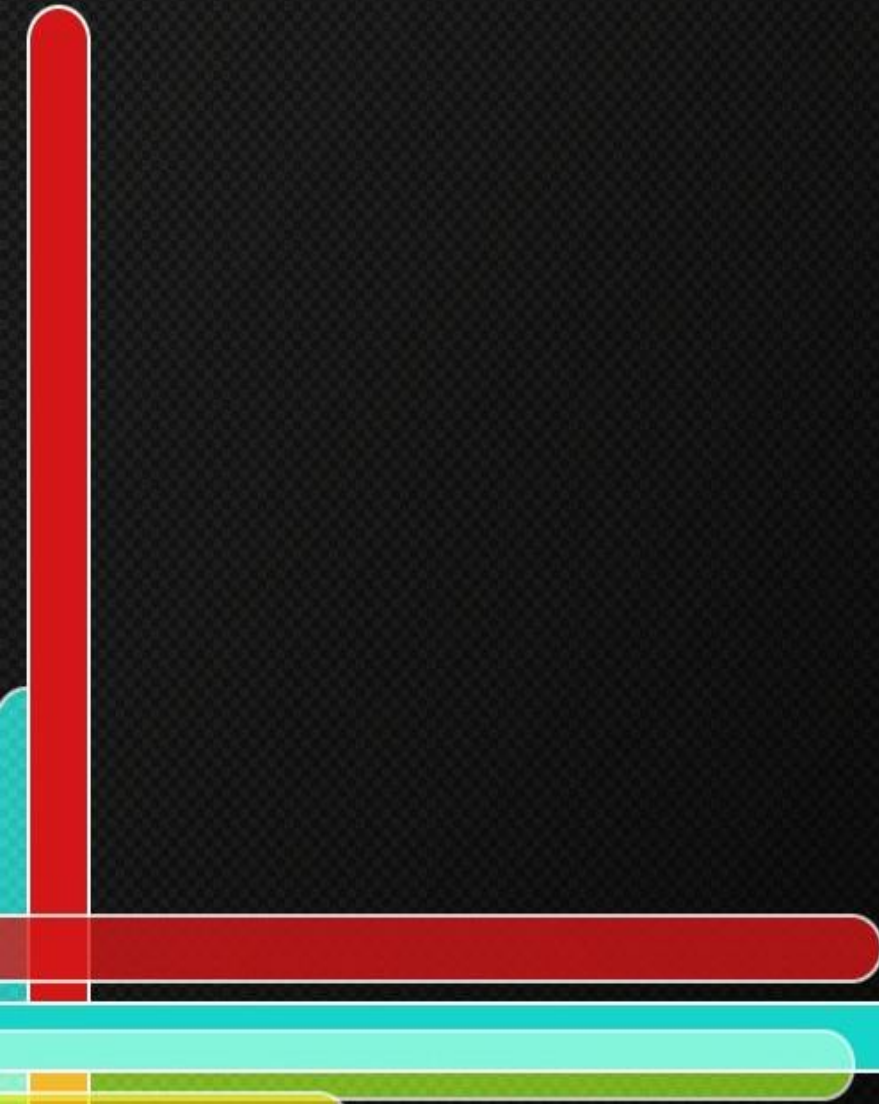
- Eigenvectors 1-3 cover the gluon uncertainty. They also contribute to \bar{u} , \bar{d} uncertainty.
- Eigenvector 1 saturates the uncertainty for most of the $gg \rightarrow H$ range.



u, d quark uncertainties are more distributed



Comparisons of CMC and META approaches



Progress in developing the combination procedure

Two methods for combination of PDFs were extensively compared, with promising results:

1. Meta-parametrizations + MC replicas + Hessian data set diagonalization

(J. Gao, J. Huston, P. Nadolsky, 1401.0013)

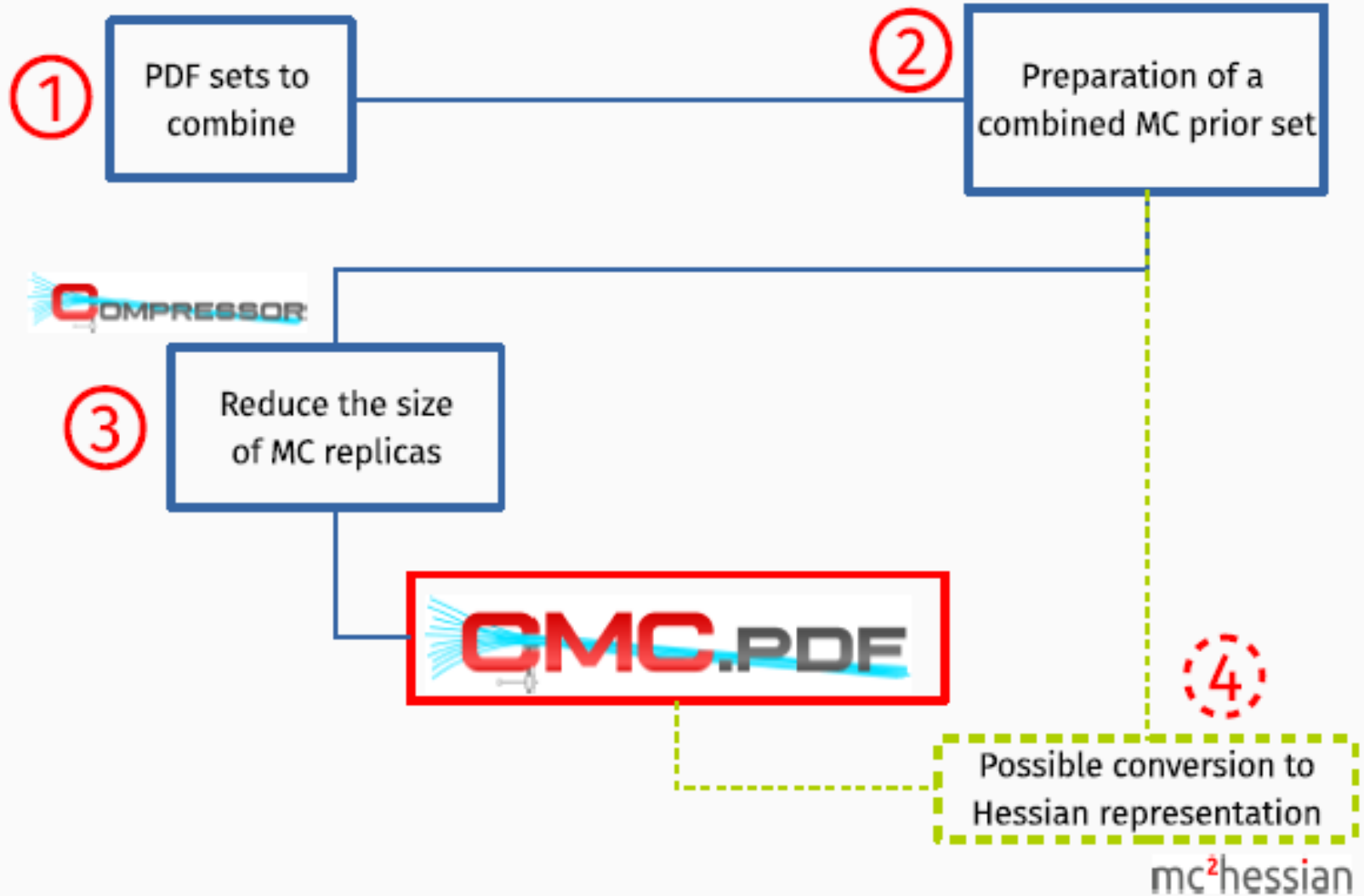
2. Compression of Monte-Carlo replicas

(Carazza, Latorre, Rojo, Watt, 1504:06469)

Both procedures start by creating a combined ensemble of MC replicas from all input ensembles (G. Watt, R. Thorne, 1205.4024; S. Forte, G. Watt, 1301.6754). They differ at the second step of reducing a large number of input MC replicas (~ 300) to a smaller number for practical applications (13-100 in the META approach; 40 in the CMC approach). The core question is how much input information to retain in the reduced replicas in each Bjorken- x region.

CMC PDFs

S. Carrazza, Feb. 2015



We define **statistical estimators** for the MC prior set:

1. **moments:** central value, variance, skewness and kurtosis
2. **statistical distances:** the Kolmogorov distance
3. **correlations:** between flavors at multiple x points

These estimators are then **compared** to subsets of replicas **interactively** driven by an *error function*, i.e.

$$\text{ERF}_{\text{tot}} = \sum_n \frac{1}{N_n} \sum_i \left(\frac{C_i^{(n)} - O_i^{(n)}}{O_i^{(n)}} \right)^2$$

where n runs over the number of statistical estimators and

- N_i is a normalization factor extracted from random realizations
- $O_i^{(n)}$ is the value of the estimator for the prior
- $C_i^{(n)}$ is the corresponding value for the compressed set



Benchmark comparisons of CMC and META PDFs

CMC ensembles with 40 replicas and META ensembles with 40-100 replicas are compared with the full ensembles of 300-600 MC replicas.

Accuracy of both combination procedures is already competitive with the 2010 PDF4LHC procedure, can be further fine-tuned by adjusting the final number of replicas.

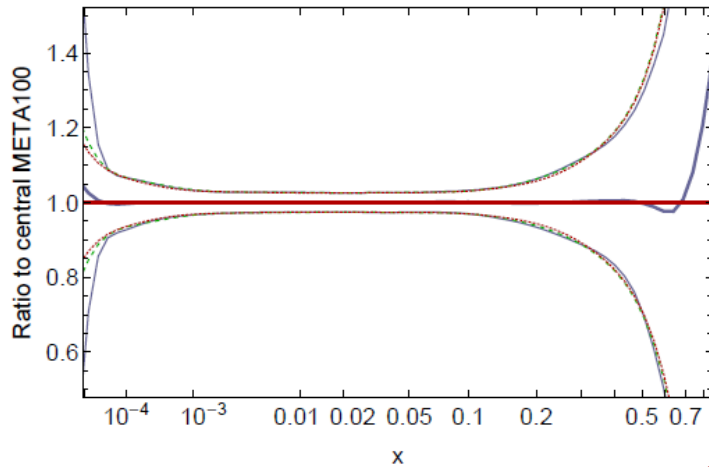
Error bands:

In the (x, Q) regions covered by the data, the agreement of 68%, 95% c.l. intervals is excellent. The definition of the central PDFs and c.l. intervals is ambiguous in extrapolation regions, can differ even within one approach. E.g., differences between mean, median, mode “central values”.

Reduction, META ensemble: 600 \rightarrow 100 \rightarrow 60 error sets

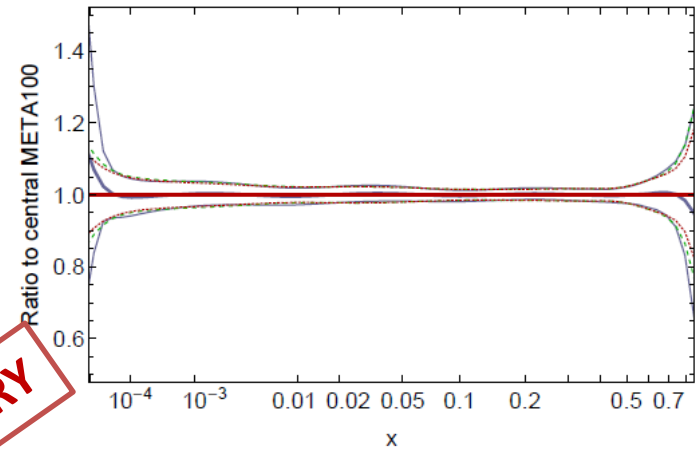
$g(x, Q)$ at $Q=8$ GeV at 68% c.l.

META600 (solid), META100 (dashed), META60 (dotted)



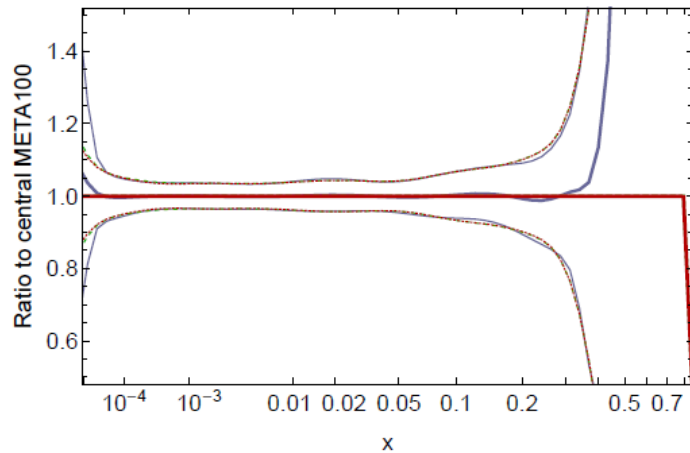
$u(x, Q)$ at $Q=8$ GeV at 68% c.l.

META600 (solid), META100 (dashed), META60 (dotted)



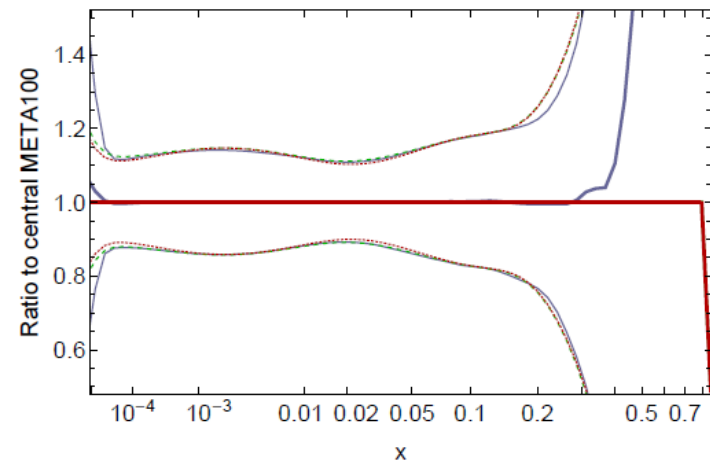
$\bar{d}(x, Q)$ at $Q=8$ GeV at 68% c.l.

META600 (solid), META100 (dashed), META60 (dotted)



$\bar{s}(x, Q)$ at $Q=8$ GeV at 68% c.l.

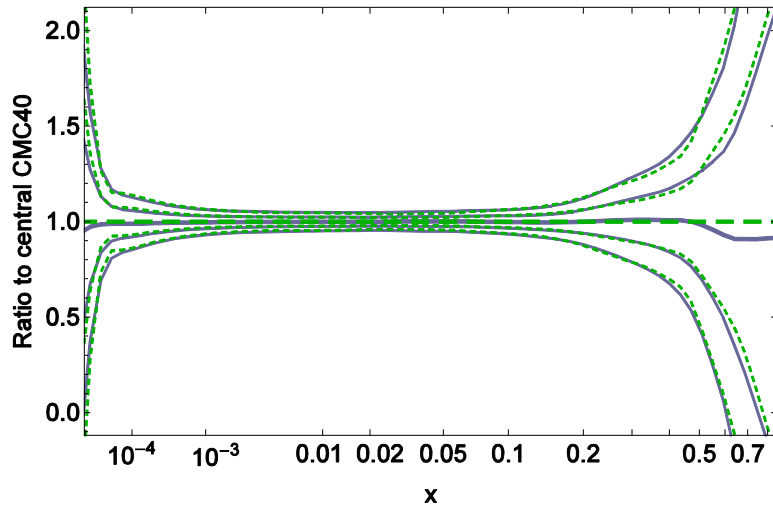
META600 (solid), META100 (dashed), META60 (dotted)



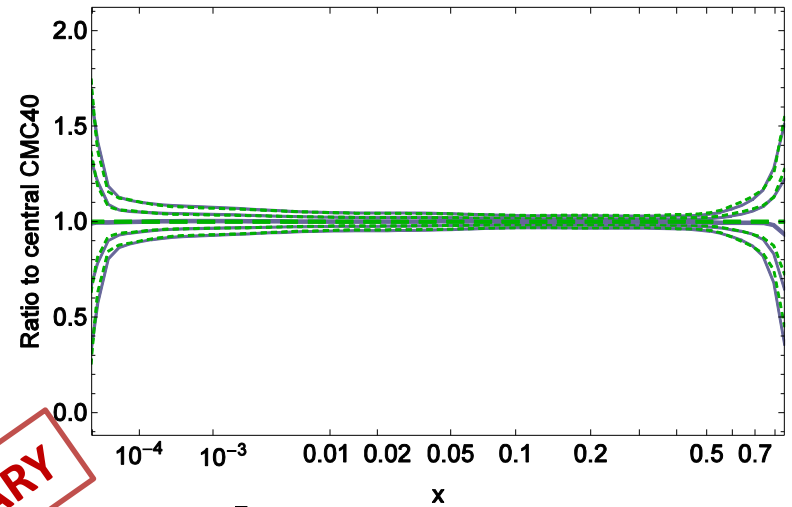
PRELIMINARY

Reduction, CMC ensemble: 300 \rightarrow 40 replicas

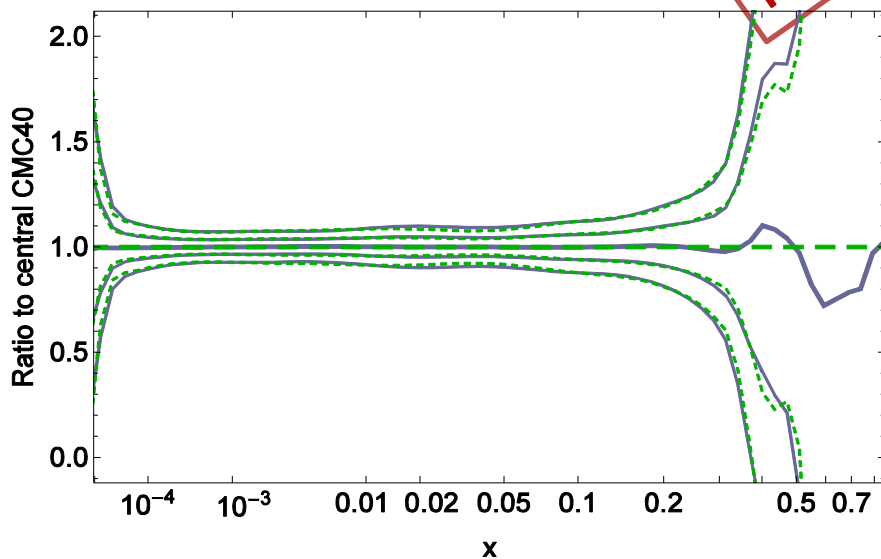
$g(x,Q)$ at $Q=8$ GeV at 1σ and 2σ
CMC40 (dashed), CMC300 (solid)



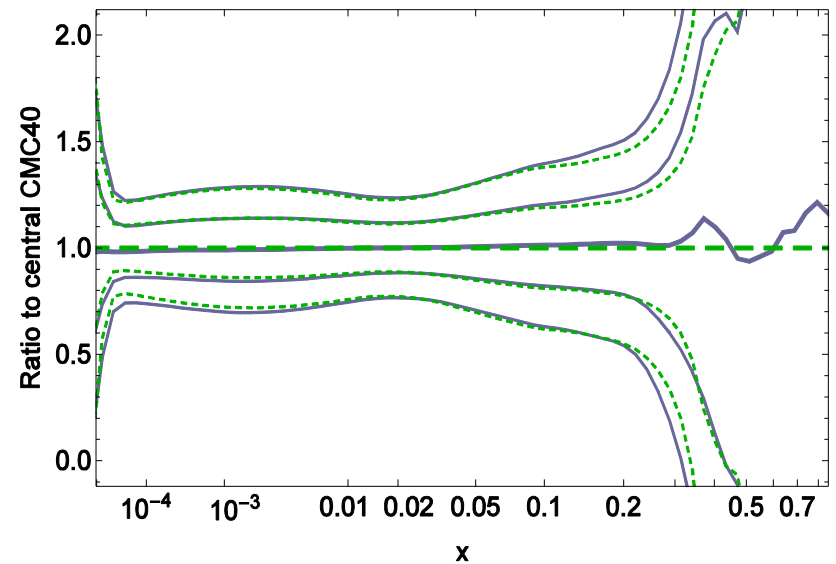
$u(x,Q)$ at $Q=8$ GeV at 1σ and 2σ
CMC40 (dashed), CMC300 (solid)



$d(x,Q)$ at $Q=8$ GeV at 1σ and 2σ
CMC40 (dashed), CMC300 (solid)



$\bar{s}(x,Q)$ at $Q=8$ GeV at 1σ and 2σ
CMC40 (dashed), CMC300 (solid)



PRELIMINARY

Benchmark comparisons, general observations II

PDF-PDF correlations:

Correlations of META300 and CMC300 ensembles differ by up to ± 0.2 as a result of fluctuations in replica generation

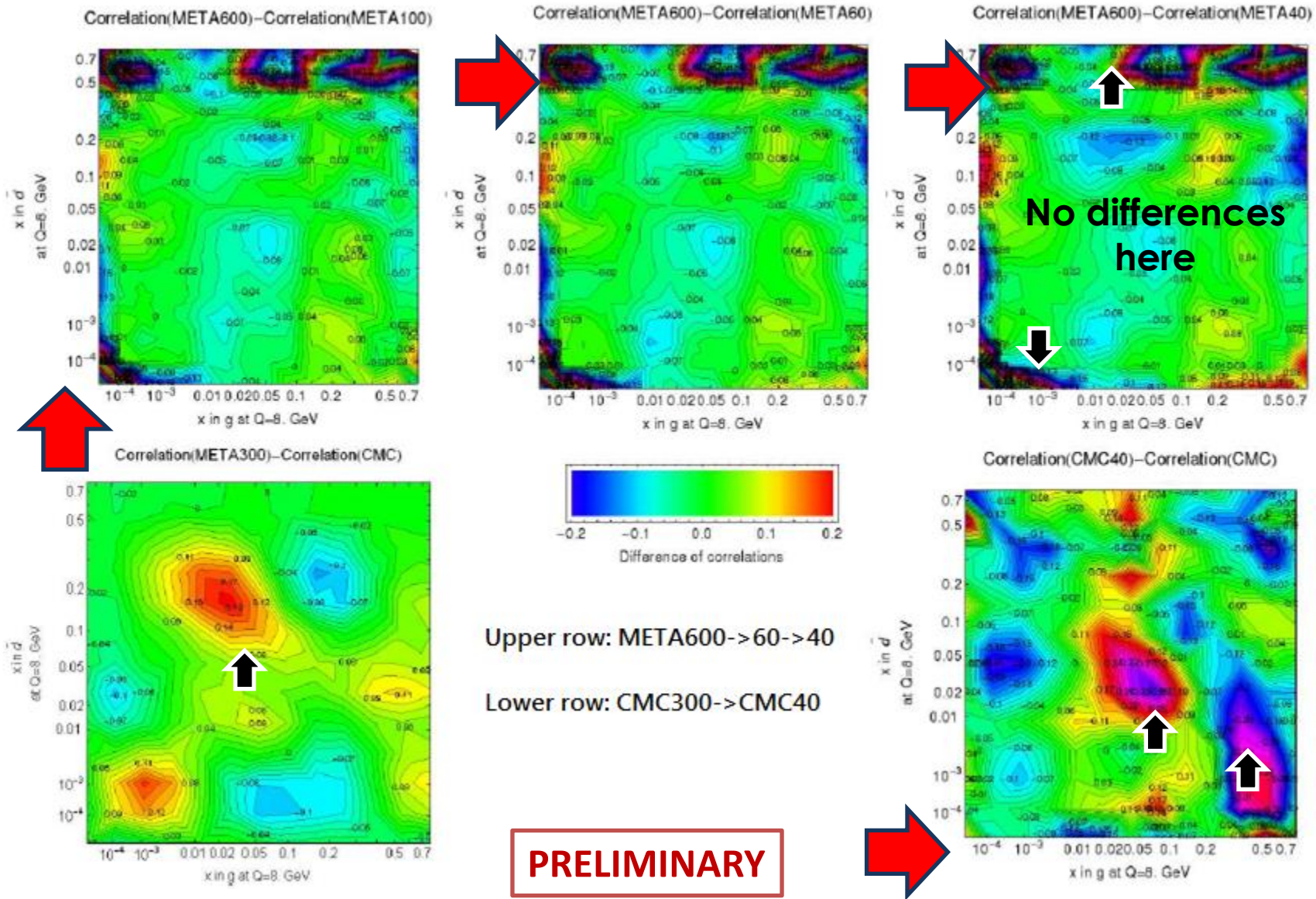
META40 PDFs faithfully reproduce PDF-PDF correlations of the META600 PDFs in the regions with data; fail to reproduce correlations in extrapolation regions \Rightarrow *next slide, upper row*

CMC40 PDFs better reproduce correlations of CMC300 in extrapolation regions; lose more accuracy in (x, Q) regions with data, but still within acceptable limits \Rightarrow *next slide, lower row*

These patterns of correlations persist at the initial scale

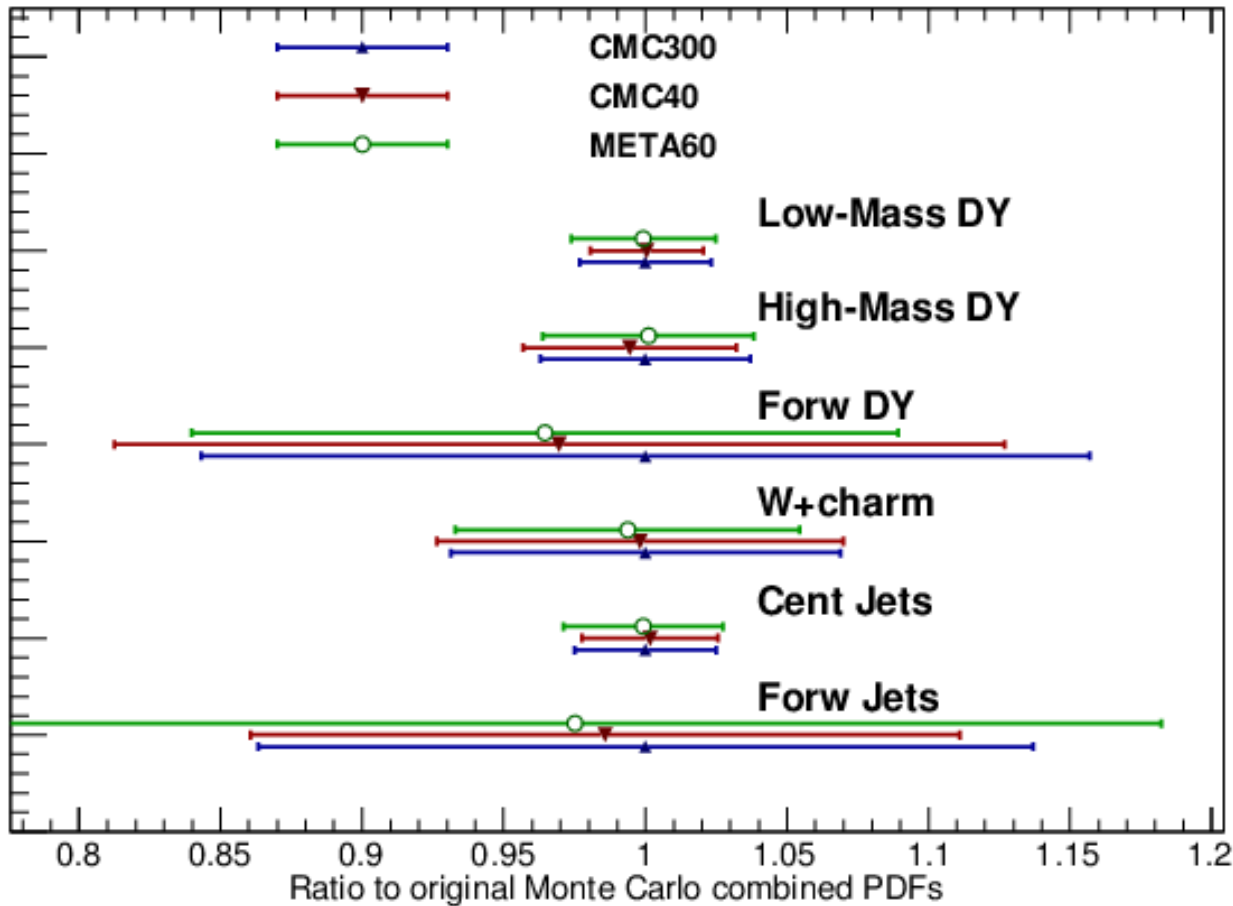
$Q_0 = 8$ GeV as well as at EW scales

PDF-PDF correlation, example: $\bar{d}(x, Q)$ vs $g(x, Q)$ at $Q = 8 \text{ GeV}$



Agreement at the level of benchmark cross sections

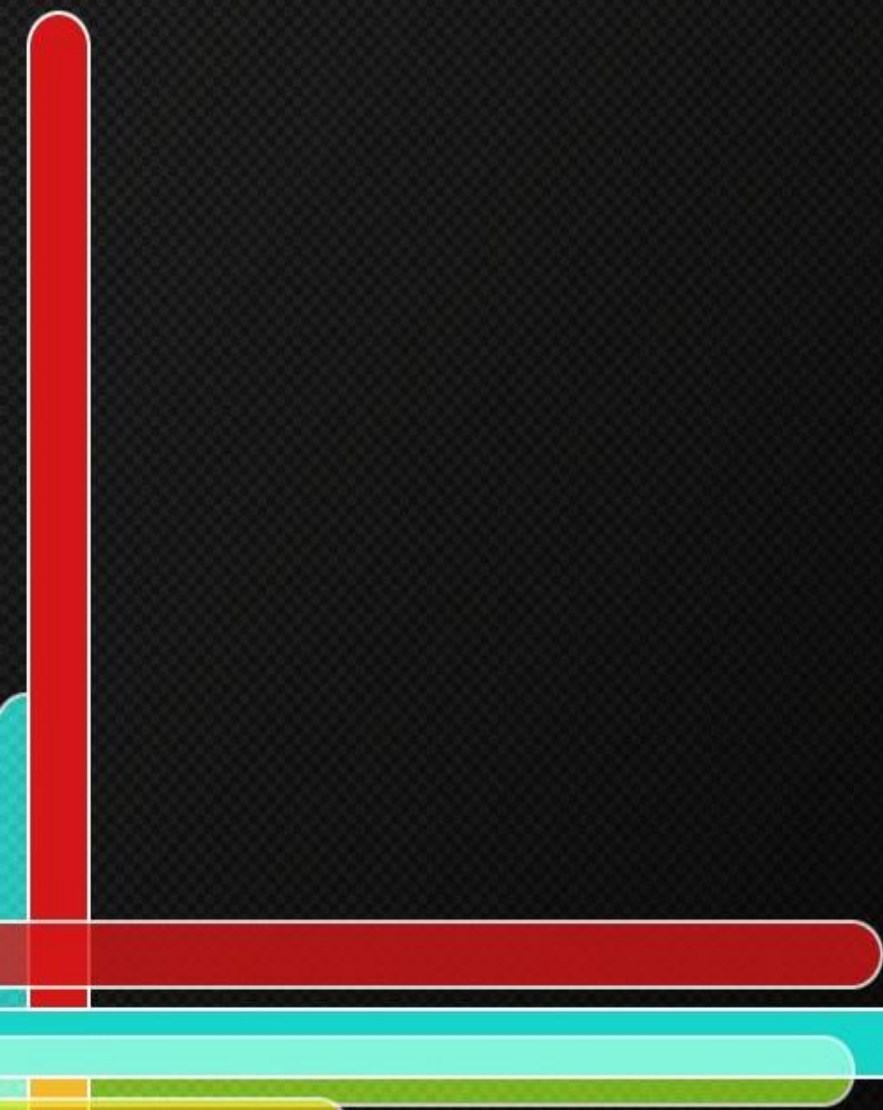
LHC 7 TeV, $\alpha_s=0.118$, NLO



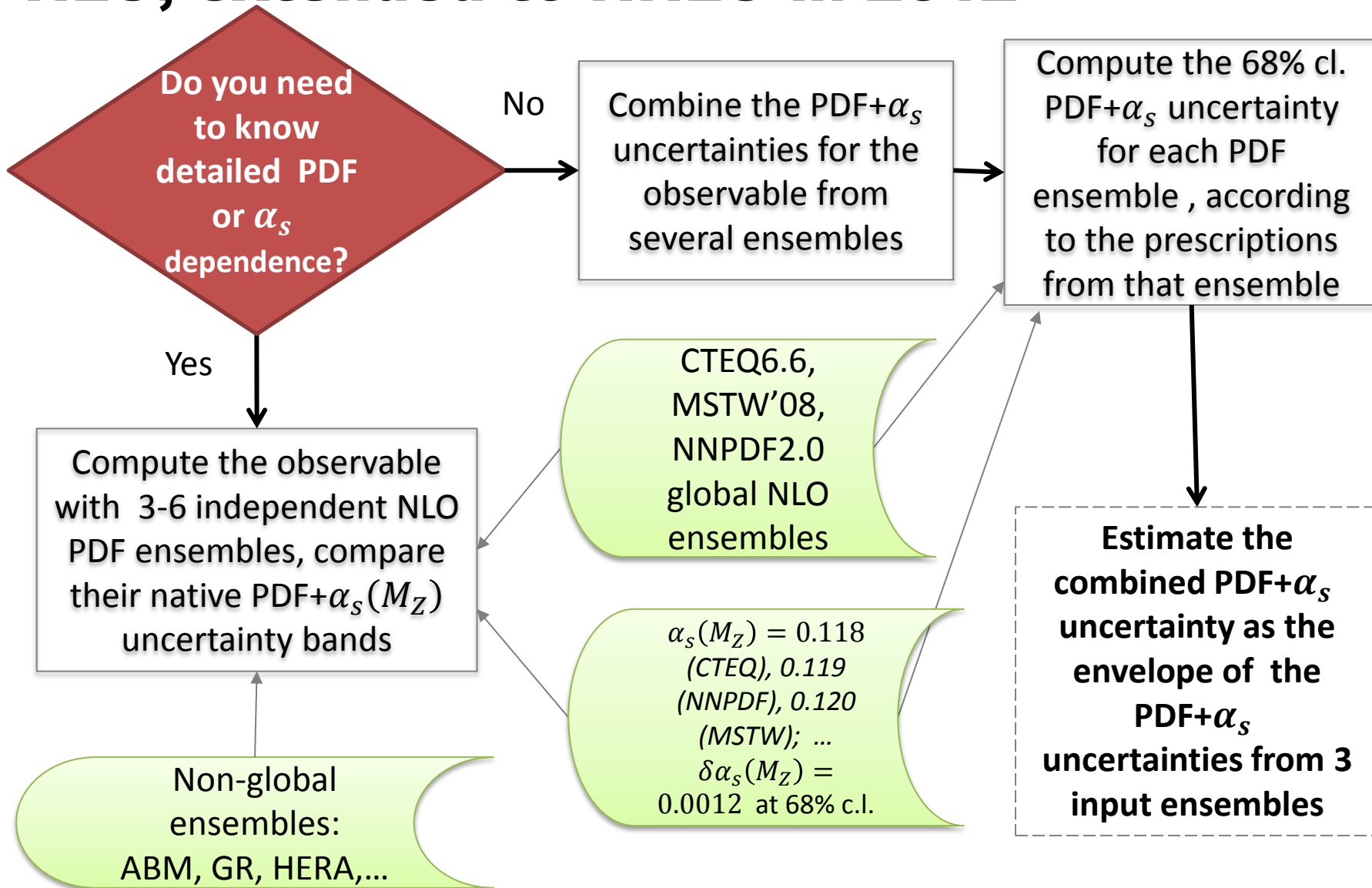
CMC-META benchmark cross sections are consistent in the x regions constrained by data

There are moderate differences in extrapolation regions. Either reduced ensemble only partly captures non-Gaussianity of the full MC ensemble at such x

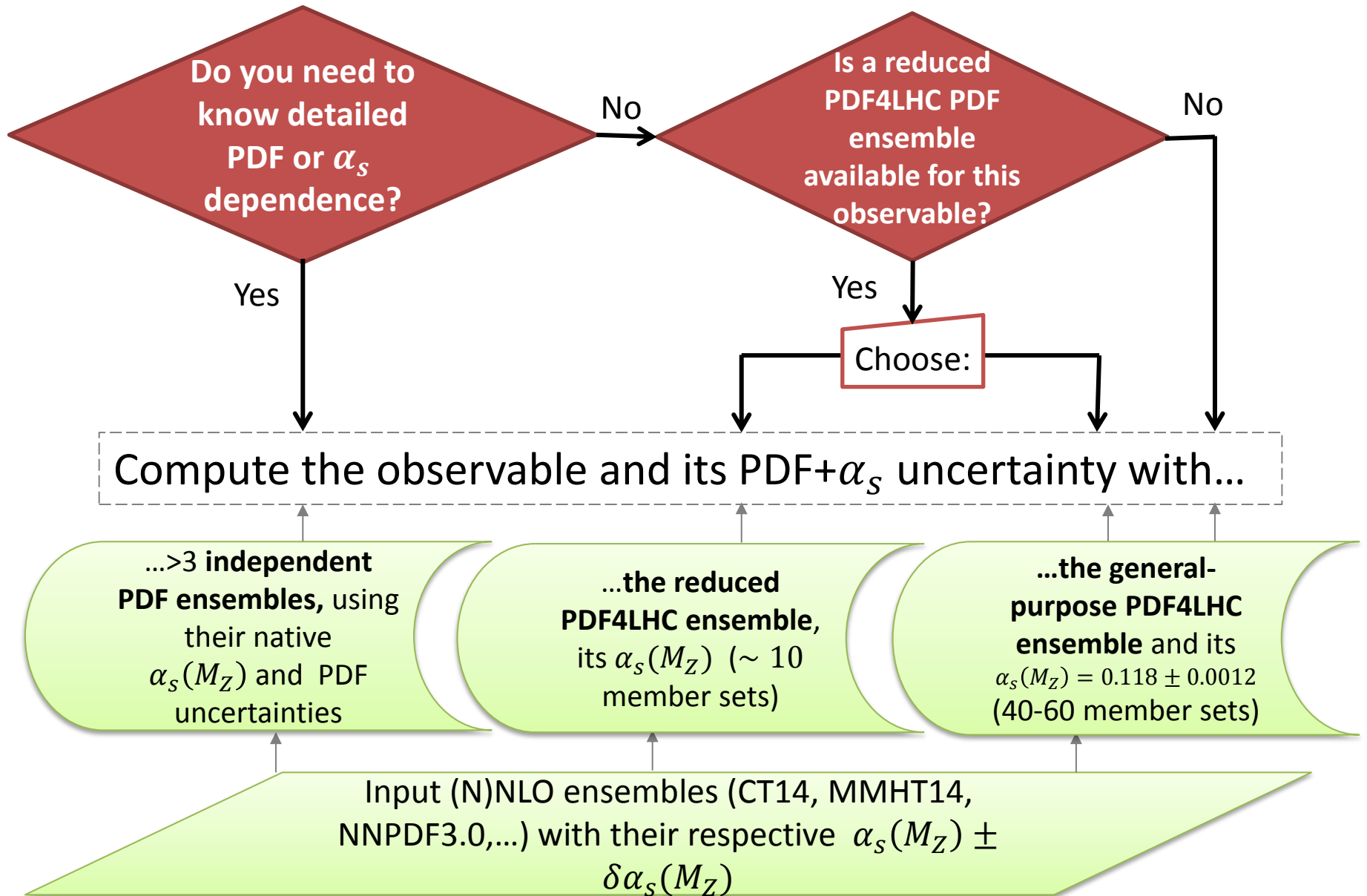
Blueprint for the 2015 PDF4LHC prescription



2010 PDF4LHC recommendation for an LHC observable: NLO; extended to NNLO in 2012



2015: A concept for a new PDF4LHC recommendation



This procedure applies both at NLO and NNLO

Combination of the PDFs into the future PDF4LHC ensemble

PDFs from several groups are combined into a PDF4LHC ensemble of error PDFs **before** the LHC observable is computed. This simplifies the computation of the PDF+ α_s uncertainty and will likely cut down the number of the PDF member sets and the CPU time needed for simulations.

The same procedure is followed at NLO and NNLO. The combination was demonstrated to work for global ensembles (CT, MSTW, NNPDF). It still needs to be generalized to allow inclusion of non-global ensembles.

The PDF uncertainty at 68% c.l. is computed from error PDFs at central $\alpha_s(M_Z)$.

Two additional error PDFs are provided with either PDF4LHC ensemble to compute the α_s uncertainty using $\alpha_s(M_Z) = 0.118 \pm 0.0012$ at the 68% c.l.