Discussion on Jet Vetoes/Binning, Uncertainties, Resummation

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Les Houches, June, 2013

Discussion Items

(Likely incomplete) list of things to discuss

- Uncertainties in jet binning
- Recombining bins
- Fixed Order
- Resummation (0-jet excl., 1-jet incl., 1-jet excl.)
- Experimental implementation of nontrivial correlations
- Clustering uncertainties ($\ln R$ effects)

Perturbative Structure of Jet Cross Sections

$$\sigma_{\text{total}} = \underbrace{\int_0^{p^{\text{cut}}} \mathrm{d}p \, \frac{\mathrm{d}\sigma}{\mathrm{d}p}}_{\sigma_0(p^{\text{cut}})} + \underbrace{\int_{p^{\text{cut}}}^{\infty} \mathrm{d}p \, \frac{\mathrm{d}\sigma}{\mathrm{d}p}}_{\sigma_{\geq 1}(p^{\text{cut}})}$$

$$\sigma_{\text{total}} = 1 + \alpha_s + \alpha_s^2 + \cdots$$

$$\sigma_{\geq 1}(p^{\text{cut}}) = \alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \cdots$$

$$\sigma_0(p^{\text{cut}}) = \sigma_{\text{total}} - \sigma_{\geq 1}(p^{\text{cut}})$$
$$= \left[1 + \alpha_s + \alpha_s^2 + \cdots\right] - \left[\alpha_s(L^2 + \cdots) + \alpha_s^2(L^4 + \cdots) + \cdots\right]$$

where $L = \ln(p^{\text{cut}}/Q)$

- Logarithms are important for $p^{\text{cut}} \ll Q \sim$ hard-interaction scale
- *Same* logarithms appear in the exclusive N-jet and inclusive ($\geq$ N+1)-jet cross section (and cancel in their sum)

Theory Uncertainties in Jet Binning

$$\sigma_{\mathrm{total}} = \sigma_0(p^{\mathrm{cut}}) + \sigma_{\geq 1}(p^{\mathrm{cut}})$$

Complete description requires full theory covariance matrix for $\{\sigma_0, \sigma_{\geq 1}\}$

- Can parametrize any 2x2 cov. matrix as the sum of 100% correlated and 100% anticorrelated pieces

$$C = \begin{pmatrix} (\Delta_0^{\mathrm{y}})^2 & \Delta_0^{\mathrm{y}}\,\Delta_{\geq 1}^{\mathrm{y}} \\ \Delta_0^{\mathrm{y}}\,\Delta_{\geq 1}^{\mathrm{y}} & (\Delta_{\geq 1}^{\mathrm{y}})^2 \end{pmatrix} + \begin{pmatrix} \Delta_{\mathrm{cut}}^2 & -\Delta_{\mathrm{cut}}^2 \\ -\Delta_{\mathrm{cut}}^2 & \Delta_{\mathrm{cut}}^2 \end{pmatrix}$$

So far just math, but very useful/convenient for physical interpretation

- Absolute "yield" uncertainty is fully correlated between bins
  - $\Delta_{\mathrm{total}} \equiv \Delta_{\geq 0}^{\mathrm{y}} = \Delta_0^{\mathrm{y}} + \Delta_{\geq 1}^{\mathrm{y}}$
- "Migration" unc. due to binning must drop out when summing $\sigma_0 + \sigma_{\geq 1}$
  - Δ_{cut} associated with uncertainties in p^{cut} log series

Theory Uncertainties in Jet Binning

More general: Consider a single binning (exclusive) cut

$$\sigma_{\geq N} = \sigma_N(\text{excl. cut}) + \sigma_{\geq N+1}(\text{inverse excl. cut})$$

$$C = \begin{pmatrix} (\Delta^{y}_{N})^2 & \Delta^{y}_{N}\,\Delta^{y}_{\geq N+1} \\ \Delta^{y}_{N}\,\Delta^{y}_{\geq N+1} & (\Delta^{y}_{\geq N+1})^2 \end{pmatrix} + \begin{pmatrix} \Delta^2_{\mathrm{cut}} & -\Delta^2_{\mathrm{cut}} \\ -\Delta^2_{\mathrm{cut}} & \Delta^2_{\mathrm{cut}} \end{pmatrix}$$

Basic questions for theory:

- What are $\Delta^{y}_{\geq N} = \Delta^{y}_{N} + \Delta^{y}_{\geq N+1}$?
- What is Δ_{cut} ?

Answers depend on ...

- ... binning cut used (tight or loose) $\longrightarrow$ migration important or not
- ... kind of calculation we do $\longrightarrow$ fixed order or excl. resummation

Jet Binning Uncertainties at Fixed Order

If we use fixed order (assuming we can), then we might want to maintain two conditions

$$\Delta^{\rm FO}_{\geq N} \stackrel{!}{=} \Delta_{\geq N} = \Delta^{\rm y}_{N} + \Delta^{\rm y}_{\geq N+1}$$
$$(\Delta^{\rm FO}_{\geq N+1})^2 \stackrel{!}{=} \Delta^2_{\geq N+1} = (\Delta^{\rm y}_{\geq N+1})^2 + \Delta^2_{\rm cut}$$

where $\Delta^{\rm FO}_{i}$ is the fixed-order uncertainty (from scale variation or otherwise)

At large $p_T^{\rm cut}$ ($\longrightarrow$ loose binning): $\sigma_{\geq N+1} \ll \sigma_{\geq N}, \sigma_N$

- Migration effects are small and can thus be neglected

$\Rightarrow \quad \Delta_{\rm cut} = 0 \quad \Rightarrow \quad \Delta^{\rm y}_{i} = \Delta^{\rm FO}_{i}$

$\Rightarrow$ Gives common 100% correlated fixed-order scale variation

At small $p_T^{\rm cut}$ ($\longrightarrow$ tight binning)

- Logs degrade FO perturbation theory $\longrightarrow$ cannot neglect $\Delta_{\rm cut}$ anymore
- At FO there is no (simple) way to split $\Delta^{\rm FO}_{\geq N+1}$ into $\Delta^{\rm y}_{\geq N+1} \oplus \Delta_{\rm cut}$

Fixed Order Uncertainties at Small $p_T^{\rm cut}$

"ST method" [Stewart, FT, 1107.2117]

- Estimate $\Delta_{\rm cut}$ by FO scale variation of $\sigma_{\geq 1}(p^{\rm cut}) = \alpha_s L^2 + \cdots$

$\Rightarrow \Delta_{\rm cut} = \Delta^{\rm FO}_{\geq N+1}\,,\ \Delta^{\rm y}_{N} = \Delta^{\rm FO}_{\geq \rm N}$

$\Rightarrow \Delta^2_N = (\Delta^{\rm FO}_{\geq N})^2 + (\Delta^{\rm FO}_{\geq N+1})^2$

"Efficiency method" [Banfi, Salam, Zanderighi, 1203.5773]

- Estimate $\Delta_{\rm cut}$ via higher-order terms in efficiency $\sigma_N(p^{\rm cut})/\sigma_{\geq N}$

$\Rightarrow \Delta_{\rm cut} = \sigma_{\geq N}\Delta\epsilon_N\,,\ \Delta^{\rm y}_i = \epsilon_i \Delta^{\rm FO}_{\geq \rm N}$

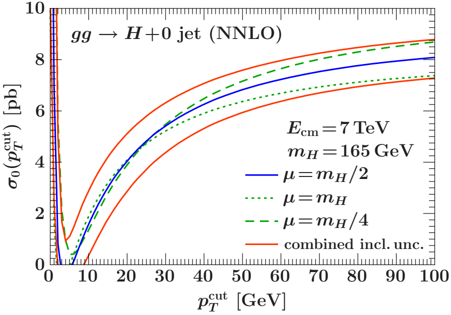
$\Rightarrow \Delta^2_N = \epsilon^2_N(\Delta^{\rm FO}_{\geq N})^2 + \sigma^2_{\geq N}(\Delta\epsilon_N)^2$

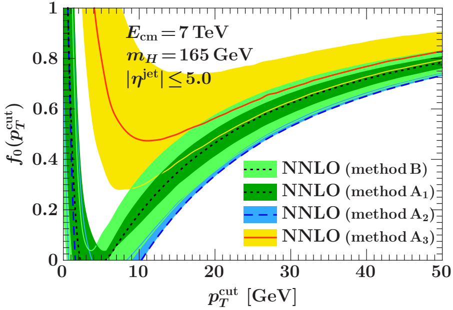
Important things to keep in mind

- In each case we are making an assumption in order to estimate $\Delta_{\rm cut}$. At large $p_T^{\rm cut}$ this doesn't matter, while at small $p_T^{\rm cut}$ either is a *much* better (safer) approximation than setting $\Delta_{\rm cut} = 0$
- Resummation of logs adds nontrivial information $\rightarrow$ allows us to improve predictions for σ_N & $\sigma_{\geq N+1}$ and to explicitly disentangle $\Delta_{\rm cut}$ and $\Delta^{\rm y}_i$

$gg \longrightarrow$ Higgs + 0 Jet at FO

blue: central scale choice
green: standard scale variation
orange: ST method to include Δ_{cut}



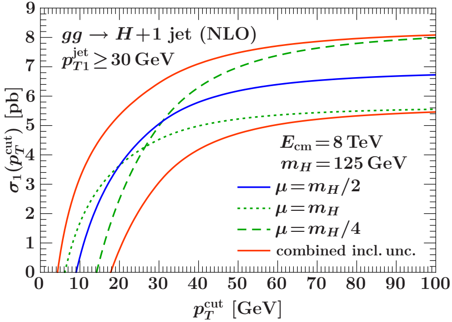


ST and efficiency methods produce very compatible results

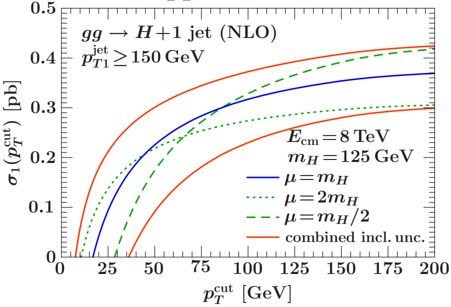
$gg \to$ Higgs + 1 Jet at FO

blue: central scale choice, green: standard scale variation
orange: ST method to include Δ_{cut}

for $p_{T1}^{jet} \geq 30$ GeV



for $p_{T1}^{jet} \geq 150$ GeV



Almost identical story repeats here

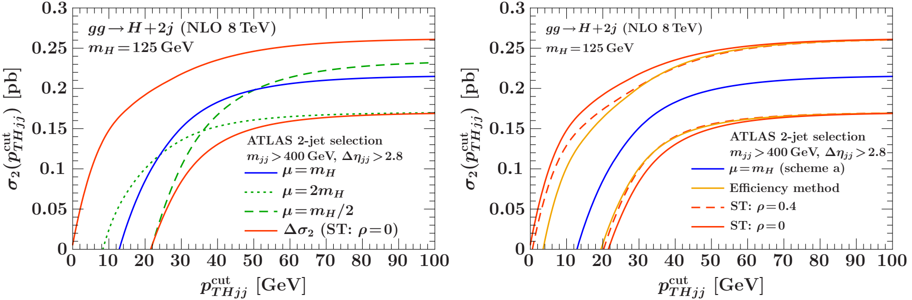
- Logs get stronger with an additional hard jet (as expected)

$gg \to$ Higgs + 2 Jet at FO

$$\sigma_{\geq 2}^{\text{VBF cuts}} = \sigma_{2}^{\text{VBF cuts}}(\text{excl. cut}) + \sigma_{\geq 3}^{\text{VBF cuts}}(\text{inverse excl. cut})$$

For ATLAS $H \to \gamma\gamma$ cut-based VBF selection (similar results for CMS)

using a cut on $p_{T,Hjj}$



blue: central scale choice
green: standard scale variation
orange: ST method to include Δ_{cut} and efficiency method

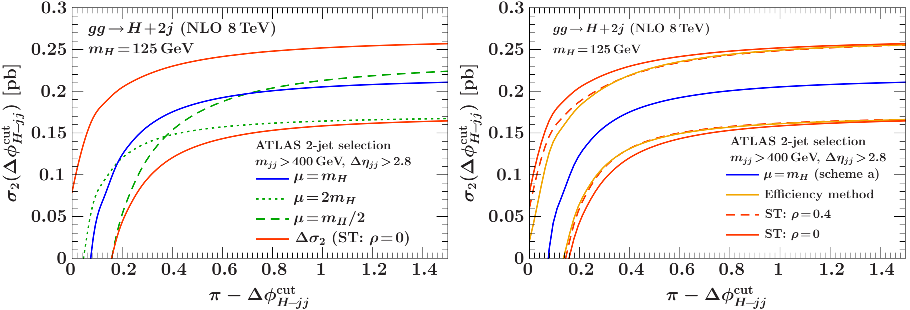
[Gangal, FT, 1302.5437]

$gg \longrightarrow$ Higgs + 2 Jet at FO

$$\sigma_{\geq 2}^{\text{VBF cuts}} = \sigma_2^{\text{VBF cuts}}(\text{excl. cut}) + \sigma_{\geq 3}^{\text{VBF cuts}}(\text{inverse excl. cut})$$

For ATLAS $H \to \gamma\gamma$ cut-based VBF selection (similar results for CMS)

using a cut on $\Delta\phi_{H-jj}$



blue: central scale choice

green: standard scale variation

orange: ST method to include Δ_{cut} and efficiency method

[Gangal, FT, 1302.5437]

Binning Uncertainties with Resummation

Including resummation in this setup is in principle straightforward.

Basic idea is to associate

- $\Delta_i^{\mathrm{y}} = \Delta_{\mu i}$
  - some "FO-type" uncertainty which reproduces FO uncertainties in limit of large p_T^{cut} (where resummation should be turned off)
- $\Delta_{\mathrm{cut}} = \Delta_{\mathrm{resum}}$
  - intrinsic "log-resummation" uncertainty relevant at small p_T^{cut} and vanishes at large p_T^{cut}

$$\Rightarrow \qquad C = \begin{pmatrix} \Delta_{\mu 0}^2 & \Delta_{\mu 0}\,\Delta_{\mu \geq 1} \\ \Delta_{\mu 0}\,\Delta_{\mu \geq 1} & \Delta_{\mu \geq 1}^2 \end{pmatrix} + \begin{pmatrix} \Delta_{\mathrm{resum}}^2 & -\Delta_{\mathrm{resum}}^2 \\ -\Delta_{\mathrm{resum}}^2 & \Delta_{\mathrm{resum}}^2 \end{pmatrix}$$

$\Rightarrow$ Precisely how to estimate these is again a separate question and depends to some extend on resummation framework

Combining Bins

Exclusive 0-jet bin

$$\Rightarrow \qquad C = \begin{pmatrix} \Delta_{\mu 0}^2 & \Delta_{\mu 0}\,\Delta_{\mu \geq 1} \\ \Delta_{\mu 0}\,\Delta_{\mu \geq 1} & \Delta_{\mu \geq 1}^2 \end{pmatrix} + \begin{pmatrix} \Delta_{0\mathrm{resum}}^2 & -\Delta_{0\mathrm{resum}}^2 \\ -\Delta_{0\mathrm{resum}}^2 & \Delta_{0\mathrm{resum}}^2 \end{pmatrix}$$

Exclusive 1-jet bin

$$\Rightarrow \qquad C = \begin{pmatrix} \Delta_{\mu 1}^2 & \Delta_{\mu 1}\,\Delta_{\mu \geq 2} \\ \Delta_{\mu 1}\,\Delta_{\mu \geq 2} & \Delta_{\mu \geq 2}^2 \end{pmatrix} + \begin{pmatrix} \Delta_{1\mathrm{resum}}^2 & -\Delta_{1\mathrm{resum}}^2 \\ -\Delta_{1\mathrm{resum}}^2 & \Delta_{1\mathrm{resum}}^2 \end{pmatrix}$$

Basic issues: How to connect the common boundary $\sigma_{\geq 1}$

- 0-jet: $\Delta_{\geq 1} = \Delta_{\mu \geq 1} \oplus \Delta_{0\mathrm{resum}}$
- 1-jet: $\Delta_{\geq 1} = \Delta_{\geq 1}^{\mathrm{FO}}$
- Region of low $p_{T1}^{\mathrm{jet}} \ll m_H$ in 1-jet bin

Resummation for Jet p_T

work with Iain Stewart, Jon Walsh, Saba Zuberi

Work by different groups

0-jet resummation

- Banfi, Monni, Salam, Zanderighi [1203.5773, 1206.4998]
  - Use QCD NNLL resummation for p_T^H [Bozzi, Catani, Grazzini] plus necessary correction terms to go from p_T^H to p_T^{jet}
- Becher, Neubert, Rothen [1205.3806, + updating numerics]
  - Use SCET-II together with "collinear anomaly" treatment to exponentiate rapidity logarithms by hand
- Stewart, FT, Walsh, Zuberi [1206.4312, + to appear]
  - Use SCET-II together with rapidity renormalization group to resum rapidity logs

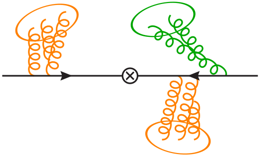
1-jet resummation

- Liu, Petriello [1210.1906, 1303.4405]
  - Resummation for large $p_{T1}^{\mathrm{jet}} \sim m_H$ using SCET-II with rapidity RGE

Factorization for Local $p_T^{\rm jet}$ Veto

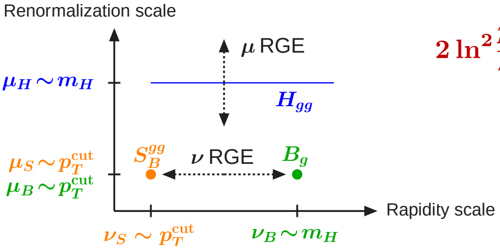
For $R^2 \ll 1$ local jet-veto measurement factorizes into simple product

$$\mathcal{M}^{\rm jet} = \mathcal{M}^{\rm jet}_{n_a}\,\mathcal{M}^{\rm jet}_{n_b}\,\mathcal{M}^{\rm jet}_{s}$$



$$\sigma_0(p_T^{\rm cut}) = H(Q,\mu)\,B^{\rm jet}(R, p_T^{\rm cut}, \mu, \nu)\,B^{\rm jet}(R, p_T^{\rm cut}, \mu, \nu)\,S^{\rm jet}(R, p_T^{\rm cut}, \mu, \nu)$$

Logarithms are split apart and resummed using coupled RGEs in μ and ν



$$2\ln^2\frac{p_T^{\rm cut}}{m_H} = 2\ln^2\frac{m_H}{\mu} + 4\ln\frac{p_T^{\rm cut}}{\mu}\ln\frac{\nu}{m_H} + 2\ln\frac{p_T^{\rm cut}}{\mu}\ln\frac{\mu\,p_T^{\rm cut}}{\nu^2}$$

Resummation Structure and Log Counting

$$\ln \sigma_0(p_T^{\rm cut}) \sim \sum_n \alpha_s^n \ln^{n+1}\frac{p_T^{\rm cut}}{m_H}\,(1+\alpha_s+\alpha_s^2+\cdots) \sim \text{LL}+\text{NLL}+\text{NNLL}+\cdots$$

Resummation conventions:	Fixed-order corrections: matching	full FO	Resummation input: $\gamma_{H,B,S}^{\mu,\nu}$	$\Gamma_{\rm cusp}$	β
LL	1	-	-	1-loop	1-loop
NLL	1	-	1-loop	2-loop	2-loop
NLL+NLO	1	α_s	1-loop	2-loop	2-loop
NLL′+NLO	α_s	α_s	1-loop	2-loop	2-loop
NNLL+NLO	α_s	α_s	2-loop	3-loop	3-loop
NNLL+NNLO	α_s	α_s^2	2-loop	3-loop	3-loop
NNLL′+NNLO	α_s^2	α_s^2	2-loop	3-loop	3-loop
N^{3}LL+NNLO	α_s^2	α_s^2	3-loop	4-loop	4-loop

- “matching” are the singular FO corrections that act as starting/boundary conditions in the resummation
- “full FO” means adding remaining FO terms not included in the resummation

Scale Choices

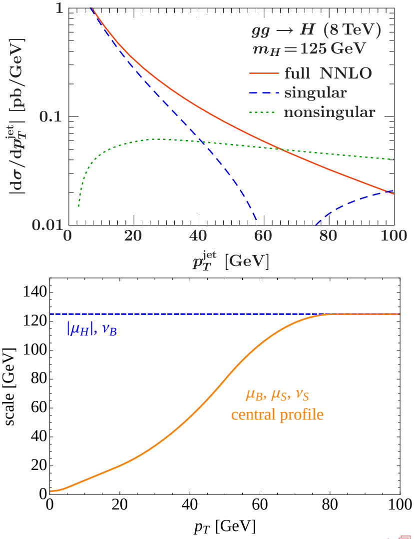
- Resummation region: Logs are resummed using canonical scaling

$$\mu_H \sim -i m_H$$
$$\mu_S \sim p_T^{\rm cut}, \nu_S \sim p_T^{\rm cut}$$
$$\mu_B \sim p_T^{\rm cut}, \nu_B \sim m_H$$

- FO region: Resummation turned off to ensure proper cancellation between singular and nonsingular terms by taking

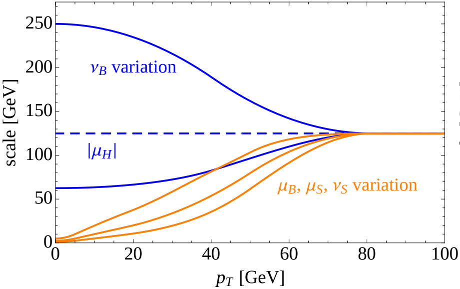
$$\mu_B, \mu_S, \nu_S, \nu_B \to \mu_{\rm FO} \sim m_H$$

- Transition region: Profiles for $\mu_B, \mu_S, \nu_B, \nu_S$ provide smooth transition from resummation to fixed-order region



Perturbative Uncertainties with Resummation

Resummation uncertainty Δ_{resum}

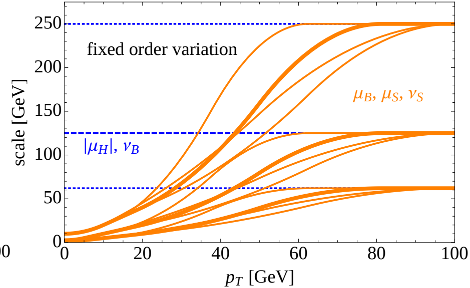


- Take maximum from separately varying all low scales (within canonical constraints)

$\Rightarrow$ Directly estimates size of logs and missing higher log terms

$\Rightarrow$ $\Delta_{\mathrm{cut}} = \Delta_{\mathrm{resum}}$

Fixed-order uncertainty Δ_μ



- Take max of collective up/down variation (+ where resum. turns off)

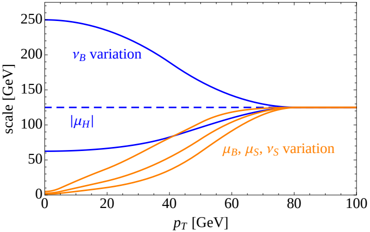
$\Rightarrow$ Equivalent to overall FO μ variation keeping logs fixed

$\Rightarrow$ Reproduces $\Delta^{\mathrm{FO}}_{\geq 0}$ for large p_T^{cut}

$\Rightarrow$ $\Delta_i^{\mathrm{y}} = \Delta_{\mu i}$

Perturbative Uncertainties with Resummation

Resummation uncertainties Δ_{resum}



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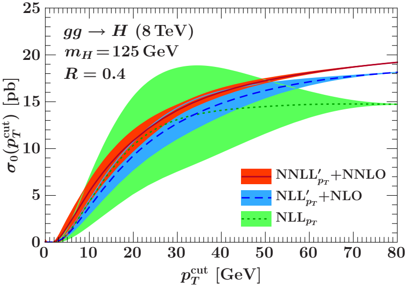
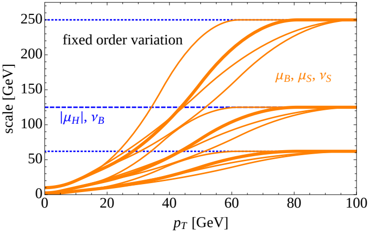


Fixed-order uncertainties $\Delta_{\mu 0}$



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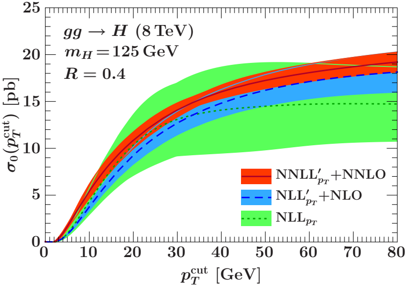
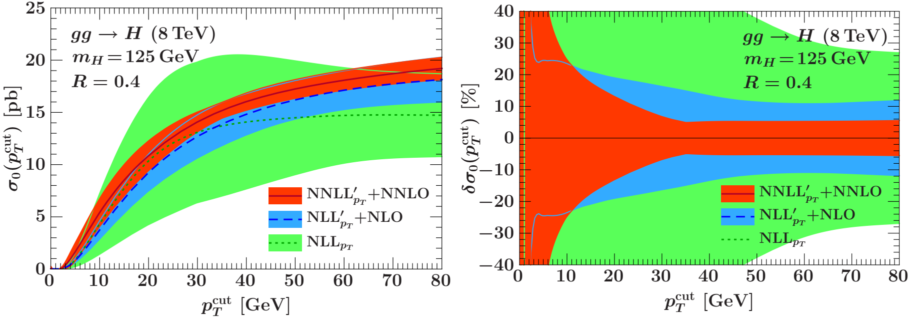


Resummed Results for Jet p_T

green: NLL_{p_T}
blue: $\mathrm{NLL}'_{p_T}+\mathrm{NLO}_0$
orange: $\mathrm{NNLL}'_{p_T}+\mathrm{NNLO}_0$



- Excellent convergence $\longrightarrow$ important check on uncertainties

$\Rightarrow$ Variation of low resummation scales $\mu_S, \mu_B, \nu_S \sim p_T^{\mathrm{cut}}$ is essential

- Additional uncertainties due to unresummed clustering logs $\ln R^2$ are not included here ($\longrightarrow$ extra slides)

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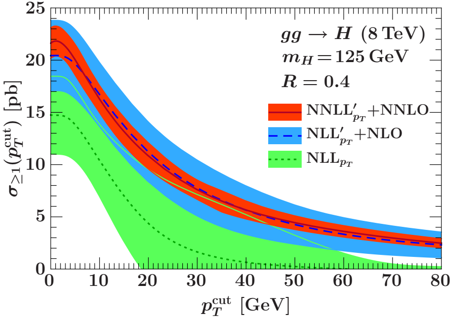

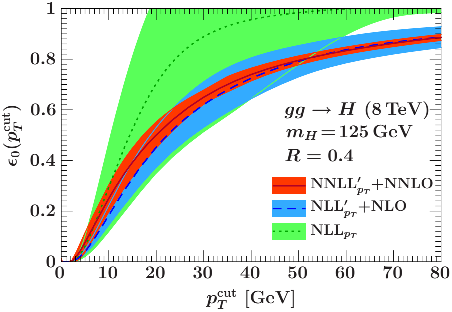

- Excellent convergence ⟶ important check on uncertainties

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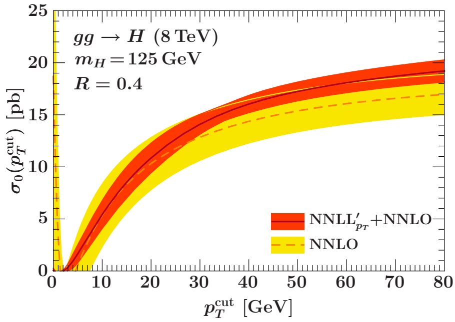
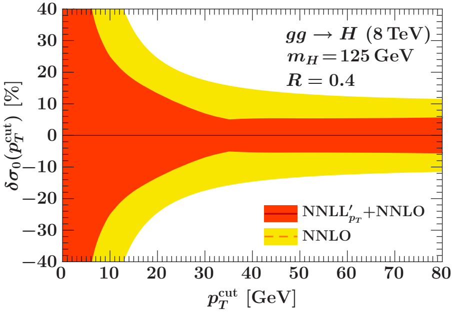
Comparison to Fixed Order

yellow: NNLO$_0$ (meaning α_s^2)
orange: NNLL$'_{p_T}$+NNLO$_0$

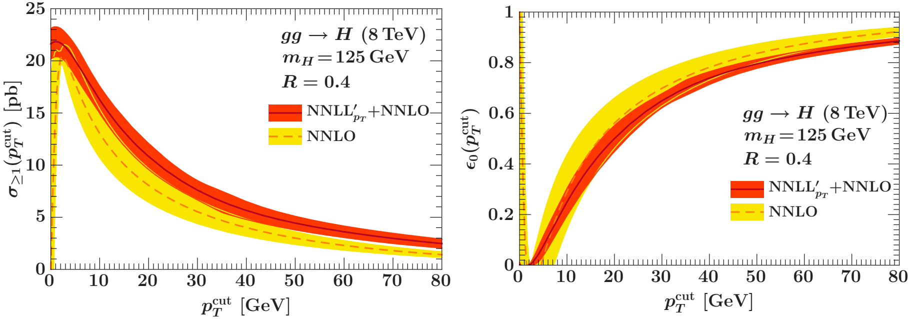




- Resummation significantly improves predictions

Comparison to Fixed Order

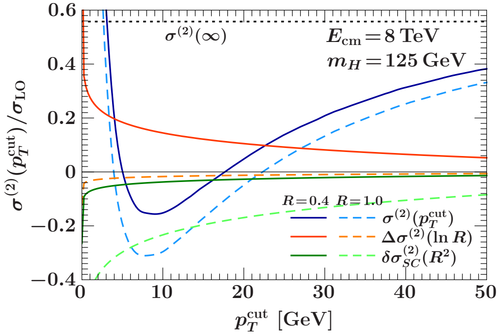
yellow: NNLO_0 (meaning α_s^2)
orange: $\mathrm{NNLL}'_{p_T}+\mathrm{NNLO}_0$



- Resummation significantly improves predictions
- Sizable increase in $\sigma_{\geq 1}$ beyond α_s^2 (NLO_1) $\longrightarrow$ compare to fixed α_s^3 (NNLO_1)

Backup Slides

Numerical Jet Algorithm Effects at NNLO



full 2-loop contribution with no veto

full 2-loop contribution with veto

clustering logs

soft-collinear mixing

For $R = 0.4$ (and also $R = 0.5$)

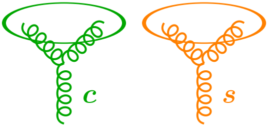
- Clustering $\ln R^2$ contributions are sizable
- Uncorrelated emission contributions (soft-collinear mixing) can safely be treated as $\mathcal{O}(R^2)$ power suppressed

$\Rightarrow$ Suggests that one should count $R^2 \sim p_T^{\rm cut}/m_H \ll 1$

Clustering Logarithms

$$\mathcal{M}^{\rm jet} = \left(\mathcal{M}^G_{n_a} + \Delta\mathcal{M}^{\rm jet}_{n_a}\right)\left(\mathcal{M}^G_{n_b} + \Delta\mathcal{M}^{\rm jet}_{n_b}\right)\left(\mathcal{M}^G_{s} + \Delta\mathcal{M}^{\rm jet}_{s}\right) + \delta\mathcal{M}^{\rm jet}$$

$\Delta\mathcal{M}^{\rm jet}_n$, $\Delta\mathcal{M}^{\rm jet}_s$: Correction from clustering of correlated emissions within soft and beam sectors



Gives rise to logs of R, leading clustering logs are

$$\frac{\Delta\sigma^{(n)}}{\sigma_B} = C_n(R)\left(\frac{\alpha_s C_A}{\pi}\right)^n \ln\frac{m_H}{p_T^{\rm cut}}\ \ln^{n-1} R^2$$

- For $R^2 \sim p_T^{\rm cut}/m_H \ \to\ \alpha_s^n L^n$ NLL series in the exponent that *cannot* be resummed at present
- Full $\alpha_s^2 C_2(R)$ term first computed by BMSZ

$\Rightarrow$ In SCET, these appear in the noncusp anomalous dimensions, allowing one to resum the $\ln(p_T^{\rm cut}/m_H)$ at NNLL$_{p_T}$ [FT, Walsh, Zuberi]

Uncertainties from Higher-Order Clustering

Since we cannot resum the clustering logs, we better estimate their size

$$\frac{1}{\sigma_B}\Delta\sigma^{(n)}(R, p_T^{\rm cut}) = C_n(R)\left[\frac{\alpha_s(p_T^{\rm cut})}{\pi}C_A \ln R^2\right]^{n-1}\left[\frac{\alpha_s(p_T^{\rm cut})}{\pi}C_A \ln\frac{m_H}{p_T^{\rm cut}}\right]$$

In this basis and for $m_H = 125\,\mathrm{GeV}$

$$\Delta\sigma^{(n)}(0.4, 25\,\mathrm{GeV})/\sigma_B = C_n(0.4)[-0.25]^{n-1}[0.22]$$

$$\Delta\sigma^{(n)}(0.7, 25\,\mathrm{GeV})/\sigma_B = C_n(0.7)[-0.10]^{n-1}[0.22]$$

Taking the leading log term only, $C_2(R)$ is a constant

$$C_2 = -2.49$$

If all C_n were of the same size, correction from next term C_3 would be

$\Delta_0^{\rm clus}(p_T^{\rm cut})$	$p_T^{\rm cut} = 25\,\mathrm{GeV}$	$p_T^{\rm cut} = 30\,\mathrm{GeV}$
$R = 0.4$:	3.6%	2.9%
$R = 0.5$:	2.1%	1.7%
$R = 0.7$:	0.5%	0.4%