Electroweak corrections to parton distributions

Preliminary results using the NNPDF methodology

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results presented on behalf of the NNPDF collaboration

Outline

1. Why electroweak corrections?
2. PDF evolution
   - Solution & Benchmark
3. Observables
   - How do the observables change by fixing PDFs?
4. Extracting PDFs from real data
   - How do the PDFs change by fixing observables?
5. Photon PDF from DIS fit
6. Reweighting the photon PDF with LHC data

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Motivation

Why electroweak corrections?

- ***A naïve argument***:

  The QED coupling α can affects processes in which QCD DGLAP is computed at **NLO** and higher orders

$$\frac{\mathcal{O}(\alpha_s^2)}{\mathcal{O}(\alpha)} \to \frac{\alpha_s^2(M_Z^2)}{\alpha(M_Z^2)} = \frac{0.1184^2}{1/127} \sim 1.78$$

- Leading order QED effects are comparable to NLO QCD corrections.

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- Leading order QED effects are comparable to NLO QCD corrections.

- ***Main motivations:***
  1. Provide a first **unbiased determination** of the **photon PDF with faithful uncertainty**.
  2. Assessment of their impact on **theoretical predictions**:
     - ⋆ EW measurements at the LHC,
     - ⋆ High-mass Drell-Yan and related searches, m_W determination, etc...
  3. MRST2004QED is available but **old and based on model assumptions**.

Technical aspects of QED corrections

Step by step: How to obtain the photon PDF.

- In order to achieve our goal we had to implement:

1. Modify **PDF evolution** (DGLAP)
   - ★ QCD (NLO/NNLO) + QED (LO)
2. Rewrite **observables** including the photon contribution
   - ★ Deep Inelastic Scattering, Drell-Yan and Jet
3. Add a new **PDF parametrization** for the **photon**
   - ★ $\gamma(x, Q^2)$ neural network, imposing PDF positivity
4. Perform a fit using the **NNPDF methodology**

Points (3) and (4) were completely written and optimized from scratch.

Outline

PDF evolution (DGLAP)

Solving the coupled evolution

- $\gamma(x, Q^2)$: photon PDF
  - **LO QED** evolution equations:

$$
\begin{aligned}
Q^2 \frac{\partial}{\partial Q^2} \gamma(x, Q^2) &= \frac{\alpha(Q^2)}{2\pi} \left[P_{\gamma\gamma}(\xi) \otimes e_\Sigma^2 \gamma\left(\frac{x}{\xi}, Q^2\right) + P_{\gamma q}(\xi) \otimes \sum_j e_j^2 q_j \left(\frac{x}{\xi}, Q^2\right) \right] \\
Q^2 \frac{\partial}{\partial Q^2} q_i(x, Q^2) &= \frac{\alpha(Q^2)}{2\pi} \left[P_{q\gamma}(\xi) \otimes e_i^2 \gamma\left(\frac{x}{\xi}, Q^2\right) + P_{qq}(\xi) \otimes e_i^2 q_i \left(\frac{x}{\xi}, Q^2\right) \right]
\end{aligned}
$$

with $e_\Sigma^2 = \sum_f^{n_f} N_c^f e_{q_f}^2$ (charges), and the momentum sum rule becomes

$$
\int_0^1 dxx \left\{ \sum_i q_i(x, Q^2) + g(x, Q^2) + \gamma(x, Q^2) \right\} = 1
$$

PDF evolution (DGLAP)

Multiple methods to solve QCD+QED evolution:

- in a special evolution basis, e.g. in Mellin space:

$$Q^2 \frac{\partial}{\partial Q^2} \underline{f}(N, Q^2) = P(N) \cdot \underline{f}(N, Q^2)$$

where $P(N)$ is the splitting function matrix in N space

$$\begin{aligned} P(N) &= \alpha_s(Q^2) P_{\rm LO}^{\rm QCD} + \alpha_s^2(Q^2) P_{\rm NLO}^{\rm QCD} + \alpha(Q^2) P_{\rm LO}^{\rm QED} + \\ &+ \mathcal{O}(\alpha\alpha_s) + \mathcal{O}(\alpha_s^3) + \mathcal{O}(\alpha^2) \end{aligned}$$

e.g. Roth, Weinzierl (hep-ph/0403200)

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- **our method:** combination of QCD and QED evolution solutions

$$f_i(N, Q^2) = \Gamma_{ik}^{\text{QCD}}(Q^2, Q_0^2) \cdot \Gamma_{kj}^{\text{QED}}(Q^2, Q_0^2) \cdot f_j(N, Q_0^2)$$

- Both methods treat the subleading terms in different ways.
- FastKernel implementation of DGLAP evolution.

Current PDF evolution (DGLAP)

- Our DGLAP properties: possibility to switch between **fixed** and **variable** flavor number schemes (**FFNS/VFNS**), running $\alpha(Q^2)$.
- **Fast Kernel** implementation in **x-space**, building the interpolation grid.

$$xN_j(x;\mu^2,\nu^2) = \sum_{k=1}^{N_{pdf}} \sum_{\alpha=1}^{N_x} \Gamma_{jk}^{\rm QCD}(x, x_\alpha|\mu^2,\mu_0^2) \left[x_\alpha N_k(x_\alpha;\mu_0^2,\nu^2)\right],$$

$$x_\alpha N_k(x_\alpha;\mu_0^2,\nu^2) = \sum_{l=1}^{N_{pdf}} \sum_{\beta=1}^{N_x} \Gamma_{kl}^{\rm QED}(x_\alpha, x_\beta|\nu^2,\nu_0^2) \left[x_\beta N_l(x_\beta;\mu_0^2,\nu_0^2)\right],$$

combining both kernels and setting $\mu = \nu = Q$ we obtain the final expression

$$xN_j(x;Q^2) = \sum_{l=1}^{N_{pdf}} \sum_{\beta=1}^{N_x} \underbrace{\Gamma_{jl}^{\rm QCD\cdot QED}(x, x_\beta|Q^2,Q_0^2)}_{\text{Fast Kernel}} \left[\underbrace{x_\beta N_l(x_\beta;Q_0^2)}_{\text{Input PDF}}\right]$$

where $\Gamma_{jl}^{\rm QCD\cdot QED}(x, x_\beta|Q^2,Q_0^2) = \sum_{k=1}^{N_{pdf}} \sum_{\alpha=1}^{N_x} \Gamma_{jk}^{\rm QCD}(x, x_\alpha|Q^2,Q_0^2)\Gamma_{kl}^{\rm QED}(x_\alpha, x_\beta|Q^2,Q_0^2)$

Impact of QED corrections to evolution

- Input PDF $\rightarrow$ Les Houches toy PDF +

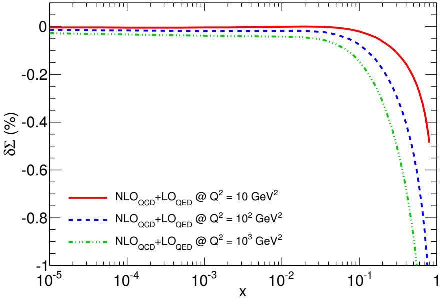
$$\hookrightarrow \boxed{x\gamma(x, Q_0^2 = 2\,\text{GeV}^2) = 0}$$

- Relative difference due to QED corrections at $Q^2 = 10, 10^2, 10^3\,\text{GeV}^2$:

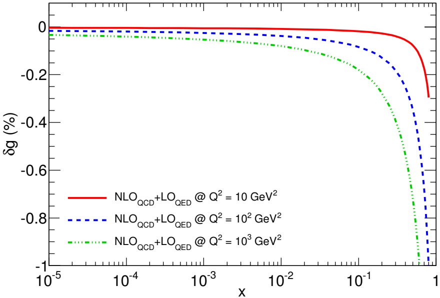
$$\delta f(x, Q^2) = \frac{f_{\text{with QED}}(x, Q^2) - f_{\text{QCD only}}(x, Q^2)}{f_{\text{with QED}}(x, Q^2)}$$

- **Singlet** and **Gluon PDFs**

QED corrections to the singlet PDF in x

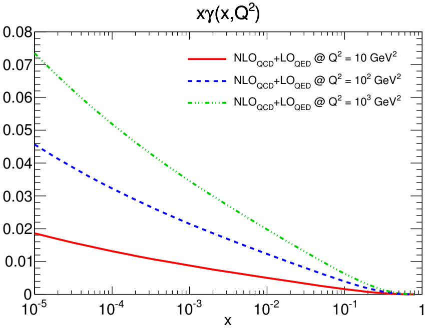


QED corrections to the gluon PDF in x



Impact of QED corrections to evolution

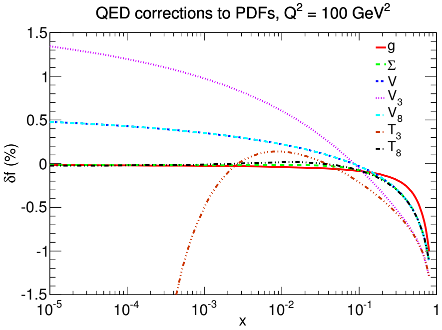
- Also for the **photon PDF!**

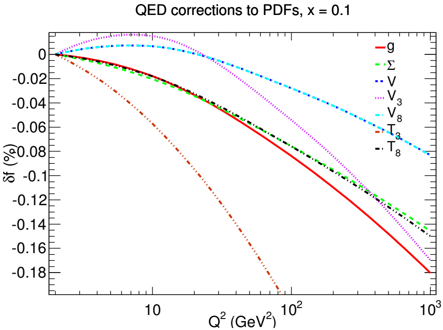


... obtained **dynamically**. $\gamma(x, Q^2)$ is minimally affected by the evolution.

Impact of QED corrections to evolution

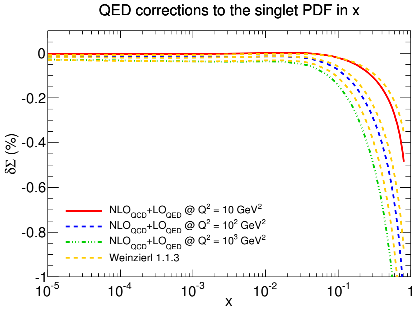
- Finally, in the **evolution basis**:

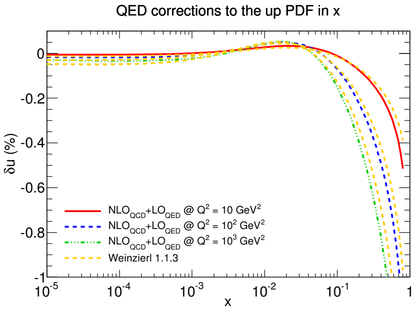




$$\begin{aligned} \Sigma &= u^+ + d^+ + s^+ \quad &|\quad V &= u^- + d^- + s^- \\ T_3 &= u^+ - d^+ \quad &|\quad V_3 &= u^- - d^- \\ T_8 &= u^+ + d^+ - 2s^+ \quad &|\quad V_8 &= u^- + d^- - 2s^- \end{aligned}$$

- Good agreement with Weinzierl 1.1.3 code[1]:
  - relative differences of the same order of magnitude
  - differences due to different solution methodology
    - ★ different subleading terms
- Here some examples for the $\Sigma(x, Q^2)$ and $u(x, Q^2)$ PDFs:



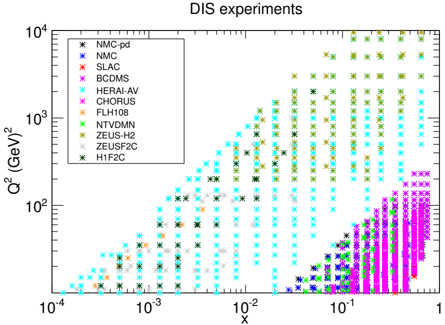


[1] hep-ph/0203112, hep-ph/0403200

Outline

Observables, current state of the art

- Observables including photon contribution due to evolution:
  - **2767 DIS data points**: e.g. $F_2^{\gamma,p}$, $F_2^{\gamma,d}$, Dimuon CC cross-section



Isospin symmetry breaking:

When activating QED corrections

$$u^p \neq d^n, \; d^p \neq u^n$$

so, e.g. $T_3^p(x, Q^2) \neq T_3^n(x, Q^2)$

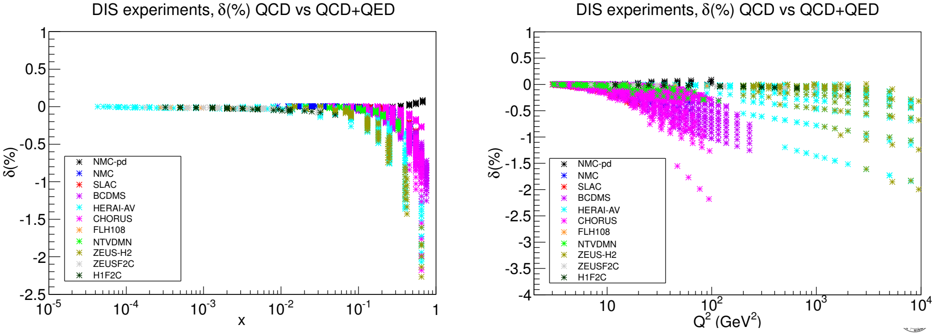
- Assume **no isospin symmetry breaking** at initial scale

$$T_3^p(x, Q_0^2) = -T_3^n(x, Q_0^2), \quad V_3^p(x, Q_0^2) = -V_3^n(x, Q_0^2)$$

- Isospin is **broken dynamically** by DGLAP evolution.

Observables, current state of the art

- Observables are codified in Fast Kernel grids
  - measure the impact on DIS data using NNPDF2.3 NLO
  - set $\gamma(x, Q_0^2) = 0$
- General behavior very similar to PDFs comparison:
  - relative differences around -1% for $x \to 1$



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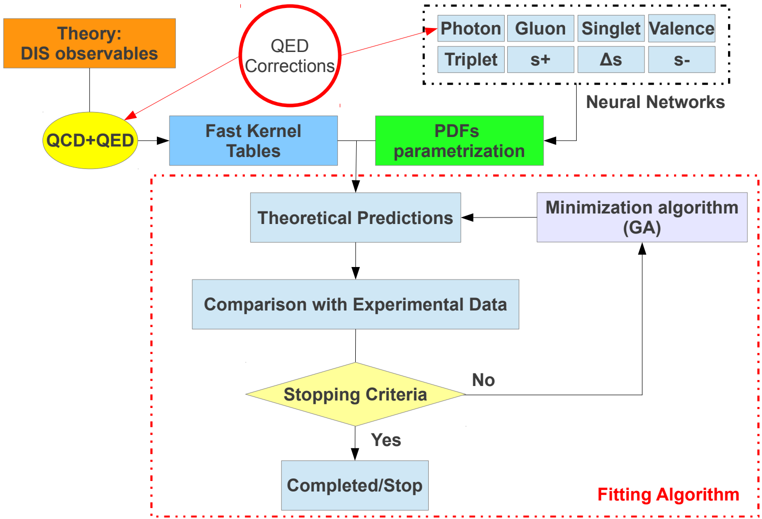
Fitting algorithm overview

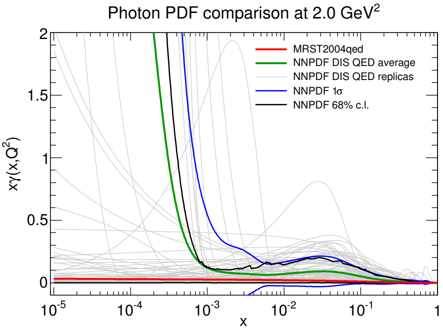


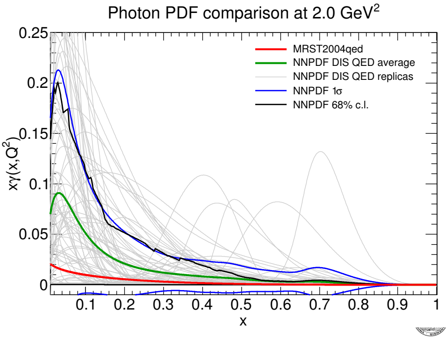
- Fit mechanism also includes momentum sum rule and positivity.

Outline

Photon PDF (preliminary)

- Performing a preliminary **DIS fit** we obtain
  - the photon PDF **extracted from data** (no toy model)
- Photon from DIS is compatible with zero with large uncertainty.
  - Z production is a good indicator





Constrain the Photon with Z production (preliminary)

- Photon PDF: impact on Z production
  - **HORACE**: Monte Carlo event generator for Drell-Yan processes including the exact 1-loop electroweak radiative corrections ($\mathcal{O}(\alpha)$)
  - **Example:** pp @ $\sqrt{s} = 14$ TeV with $|\eta^l| \leq 2.5$, $p_T^l \geq 20\,\mathrm{GeV}$

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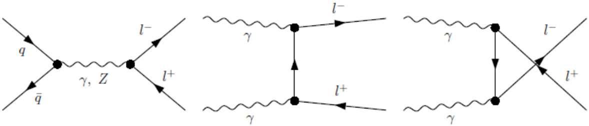
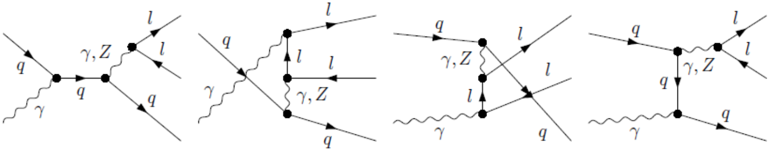
- **Born** diagrams (from arXiv:0710.1722):

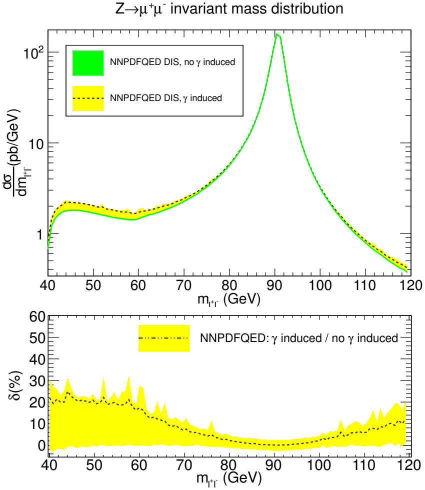


- **Photon-induced** NLO-EW process diagrams:



Constrain the Photon with Z production (preliminary)

- **Example:** $Z \to l^+ l^-$ invariant mass



- Effect of photon PDF from DIS data
  - moderate in the region of the peak
  - rapidly increases away from the peak
- Potentially huge contribution due to lack of constraints from DIS on small-x
  - ruins predictions for high m_Z/p_T^l!
- **Next step:** use W/Z production data to constraint photon PDF → use for e.g.
  - predictions for jets & Z' production

Outline

Reweighting methodology

Adding ATLAS W/Z data

- **Goal:** inclusion of LHC Drell-Yan data to constrain the photon PDF.
  - **ATLAS W/Z rapidity** (2010) $\Rightarrow$ **small-x** constraint
  - **ATLAS DY high mass** (2011) $\Rightarrow$ **central-x** constraint

Reweighting methodology

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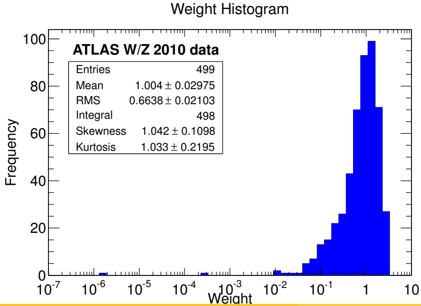
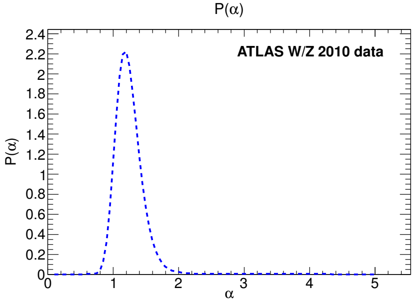
Reweighting, step by step (briefly):

1. Produce N **photon replicas from a DIS fit**
   - using MSR, dynamical stopping, positivity
2. Build a **NNPDF2.3 NLO + photon DIS** set @ $Q^2 = 2\,\text{GeV}^2$
   - recomputing evolution with QED corrections
3. **Reweight** the obtained new set with **ATLAS data**
   - 43 data points: $d\sigma/d|y_Z|$ (8), $d\sigma/d|\eta_l|$ (22) and $d\sigma/dm_{ee}$ (13)

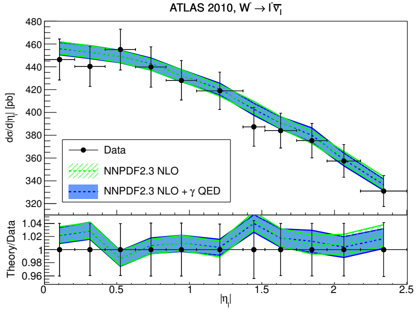
- **Input:** QED DIS fit with $N = 500$ photon replicas.
- For each replica, compute predictions using
  - **APPLgrid** for the QCD NLO.
  - **HORACE** for the photon induced (LO and NLO) contribution.
- Compute the weight of replica k (details arXiv:1108.1758)

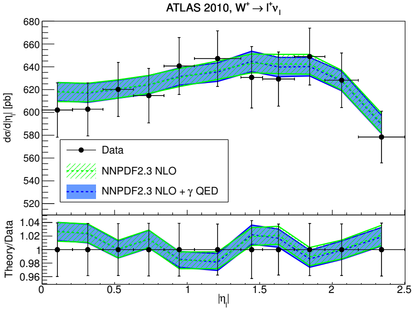
$$w_k \propto \chi_k^{n-1} e^{-\frac{1}{2}\chi_k^2}$$

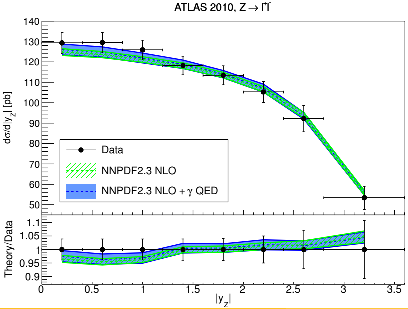




Reweighting with ATLAS W/Z data (preliminary)







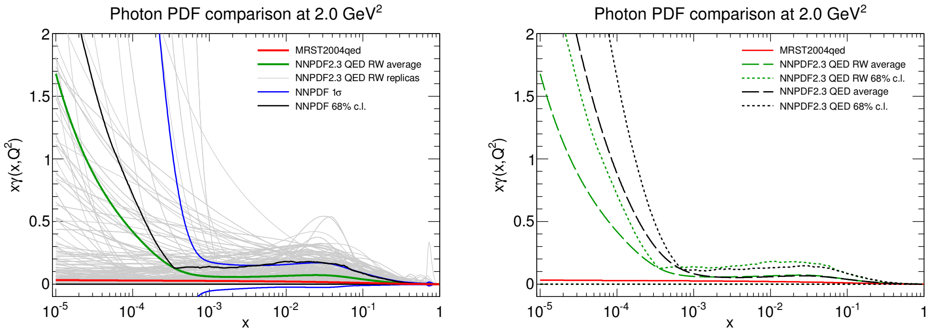
PDF set	χ^2/d.o.f.
NLO pure QCD	1.262
NLO + Photon DIS	1.178
NLO + Photon RW	1.097

- $Z \to l^+ l^-$ channel is more sensitive to the photon PDF:
  - due to $\gamma\gamma \to l^+ l^-$

Reweighting with ATLAS W/Z data (preliminary)

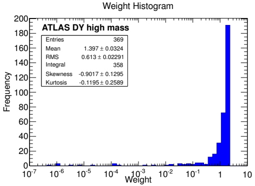
Adding ATLAS W/Z data

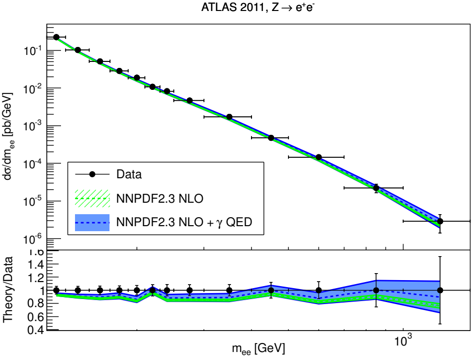
- The weighted photon PDF is constrained by the ATLAS W/Z data
  - Uncertainties are **smaller at small-x**
- From the initial $N = 500$ we obtain $N_{eff} = 345$.
- **The photon PDF after unweighting** (ATLAS W/Z data)**:**

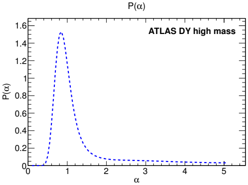


Reweighting with ATLAS DY high mass data (prel.)

- For each replica, compute with:
  - **DYNNLO** for the QCD NLO.
  - **HORACE** for the photon induced contribution.
- $N = 500 \rightarrow N_{eff} = 300$

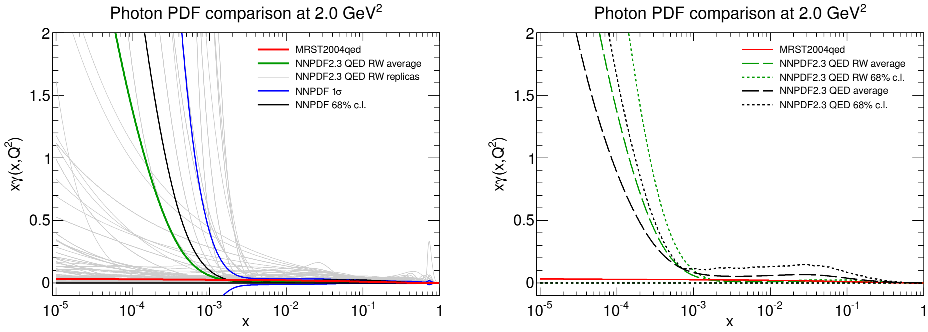






PDF set	χ^2/d.o.f.
NLO pure QCD	0.830
NLO + Photon DIS	5.715
NLO + Photon RW	0.590

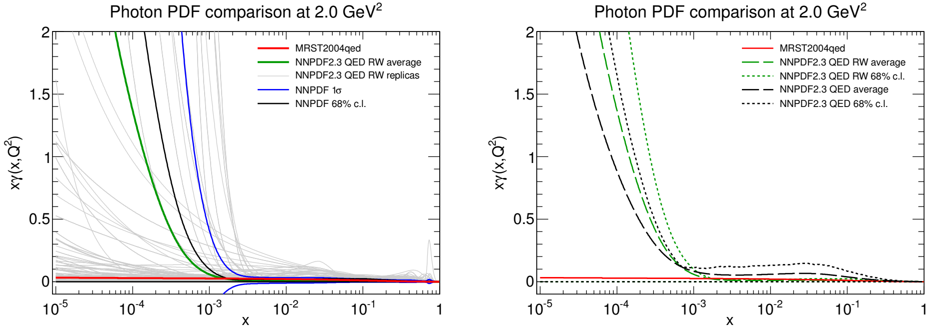
- **Uncertainties** are **reduced at central/large-*x***
- **The photon PDF after unweighting** (ATLAS DY high mass data):



Reweighting with ATLAS DY high mass data (prel.)

Adding ATLAS DY high mass data

- **Uncertainties** are **reduced at central/large-x**
- **The photon PDF after unweighting** (ATLAS DY high mass data):



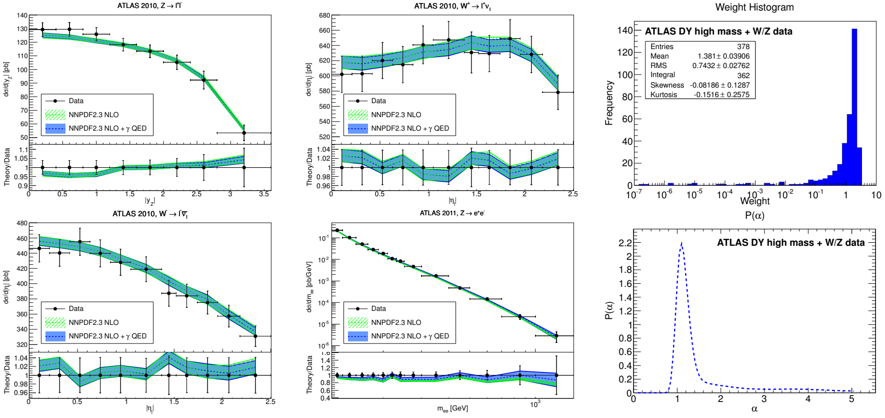
Next Step:

Combine both datasets in a single reweigthing procedure.

Combined Reweighting (preliminary)

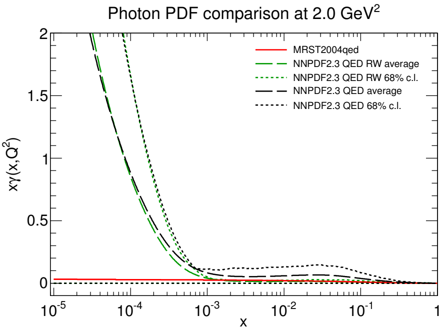
ATLAS W/Z + DY high mass data

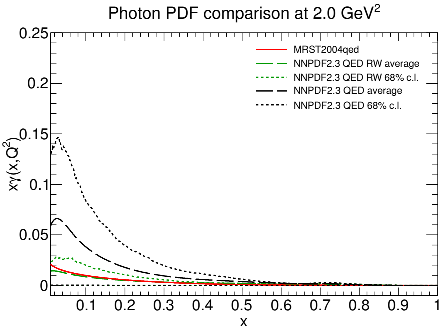
- Full reweighting with **43 data points**:
  - From $N = 500 \rightarrow N_{eff} = 280$
  - χ^2 from 2.550 to 1.012



Final photon PDF (preliminary)

- Final unweighted photon PDF
  - constrained at **small** and **central/large-x**.
  - achieved good precision for **LHC predictions**.

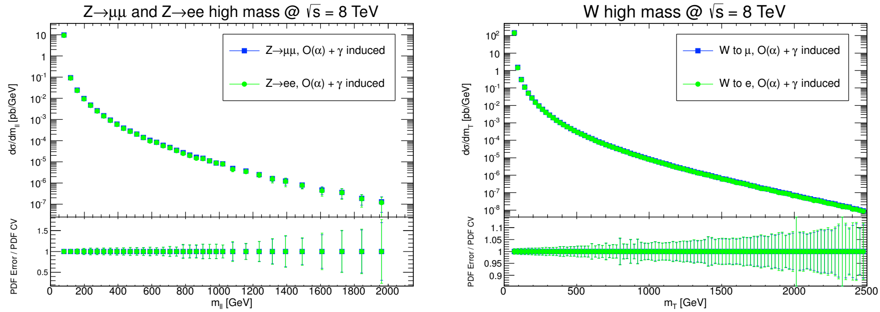




Studying predictions for BSM

Impact on new physics searches

- Simple example test with **HORACE**:
  - 10M events, $p_T^l > 25\,\text{GeV}$, $|\eta| < 2.4$, $\mathcal{O}(\alpha)$ + photon induced



- Photon PDF can improve/change limits for **BSM models.**

Outlook

- **Conclusion**
  - Extraction of a preliminary photon PDF from DIS data
  - Study the impact of a DIS photon PDF to Z production
    - **Problem:** too large uncertainties for LHC physics
    - **Solution:** reweighting with ATLAS W/Z and DY high mass data
  - Unweighting the obtained PDF set.
- **Outlook**
  - Build a NNLO QCD + LO QED fit, using the same methodology.
- **Release**
  - Release a set with QED corrections in the next LHAPDF release.

Les Houches Toy PDF

- For the benchmark we have used hep-ph/0204316

$$xu_v(x,Q_0^2) = 5.10720 \cdot x^{0.8}(1-x)^3$$

$$xd_v(x,Q_0^2) = 3.06432 \cdot x^{0.8}(1-x)^4$$

$$xg(x,Q_0^2) = 1.70000 \cdot x^{-0.1}(1-x)^5$$

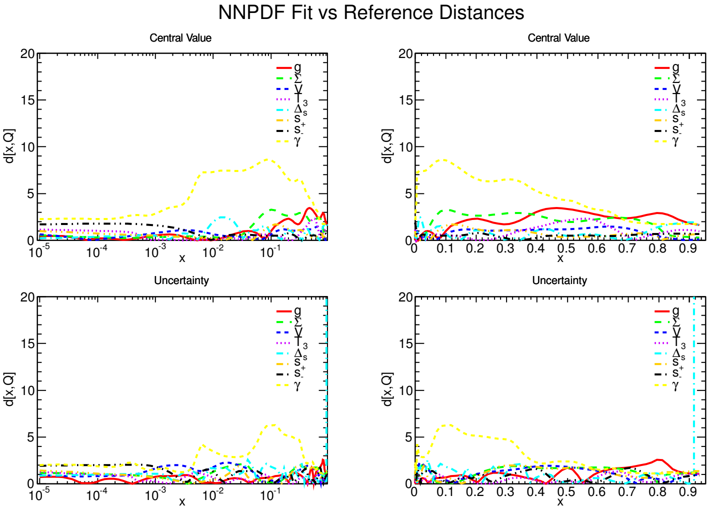
$$x\bar{d}(x,Q_0^2) = .1939875 \cdot x^{-0.1}(1-x)^6$$

$$x\bar{u}(x,Q_0^2) = (1-x)x\bar{d}(x,Q_0^2)$$

$$xs(x,Q_0^2) = x\bar{s}(x,Q_0^2) = 0.2 \cdot x(\bar{u}+\bar{d})(x,Q_0^2)$$

Photon PDF (preliminary)

- Distances between a pure QCD NLO DIS fit and the respective QED corrected fit.



PDF evolution (DGLAP)

Multiple methods to solve QCD+QED evolution:

- **(1)** in a special evolution basis, e.g. in Mellin space:

$$Q^2 \frac{\partial}{\partial Q^2} \underline{f}(N, Q^2) = P(N) \cdot \underline{f}(N, Q^2)$$

where $P(N)$ is the splitting function matrix in N space

$$P(N) = \alpha_s(Q^2) P_{\text{LO}}^{\text{QCD}} + \alpha_s^2(Q^2) P_{\text{NLO}}^{\text{QCD}} + \alpha(Q^2) P_{\text{LO}}^{\text{QED}} + \mathcal{O}(\alpha\alpha_s) + \cdots$$

e.g. Roth, Weinzierl (hep-ph/0403200)

- **(2) our method:** combination of QCD and QED evolution solutions

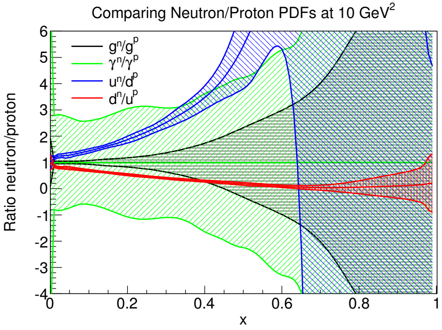
$$f_i(N, Q^2) = \Gamma_{ik}^{\text{QCD}}(Q^2, Q_0^2) \cdot \Gamma_{kj}^{\text{QED}}(Q^2, Q_0^2) \cdot f_j(N, Q_0^2)$$

Schematically $\Rightarrow$ $\begin{cases} \textbf{Method (1):} & f(N, Q^2) = \exp\left[\text{QCD} + \text{QED}\right] \cdot f(N, Q_0^2) \\ \textbf{Method (2):} & f(N, Q^2) = \exp\left[\text{QCD}\right] \cdot \exp[\text{QED}] \cdot f(N, Q_0^2) \end{cases}$

- Methods differ by subleading terms $\mathcal{O}(\alpha\alpha_s)$ (Baker-Campbell-Hausdorff)

Isospin (preliminary)

- We are able to build a **neutron PDF set** for $Q^2 > 2\,\mathrm{GeV}^2$
  - modify evolution basis $u^p \to d^n$, $d^p \to u^n$
- The ratio Neutron/Proton PDFs
  - **isospin symmetry breaking on quarks distributions**



Current PDF evolution (DGLAP)

- Our DGLAP properties: possibility to switch between **fixed** and **variable** flavor number schemes (**FFNS/VFNS**), running $\alpha(Q^2)$.
- **Fast Kernel** implementation in **x-space**, building the interpolation grid.

$$xN_j(x;\mu^2,\nu^2) = \sum_{k=1}^{N_{pdf}} \sum_{\alpha=1}^{N_x} \Gamma_{jk}^{\text{QCD}}(x, x_\alpha|\mu^2, \mu_0^2) \left[x_\alpha N_k(x_\alpha; \mu_0^2, \nu^2) \right],$$

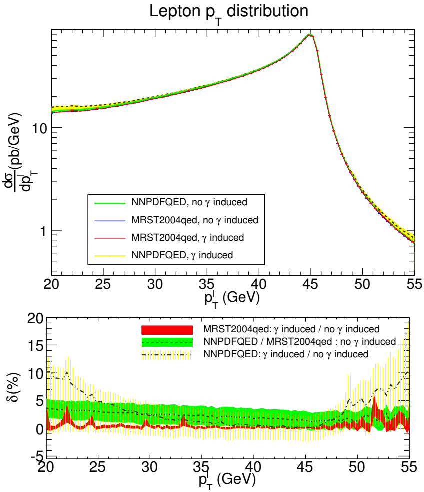
$$x_\alpha N_k(x_\alpha;\mu_0^2,\nu^2) = \sum_{l=1}^{N_{pdf}} \sum_{\beta=1}^{N_x} \Gamma_{kl}^{\text{QED}}(x_\alpha, x_\beta|\nu^2, \nu_0^2) \left[x_\beta N_l(x_\beta; \mu_0^2, \nu_0^2) \right],$$

combining both kernels and setting $\mu = \nu = Q$ we obtain the final expression

$$xN_j(x;Q^2) = \sum_{l=1}^{N_{pdf}} \sum_{\beta=1}^{N_x} \underbrace{\Gamma_{jl}^{\text{QCD}\cdot\text{QED}}(x, x_\beta|Q^2, Q_0^2)}_{\text{Fast Kernel}} \left[\underbrace{x_\beta N_l(x_\beta; Q_0^2)}_{\text{Input PDF}} \right]$$

where $\Gamma_{jl}^{\text{QCD}\cdot\text{QED}}(x, x_\beta|Q^2, Q_0^2) = \sum_{k=1}^{N_{pdf}} \sum_{\alpha=1}^{N_x} \Gamma_{jk}^{\text{QCD}}(x, x_\alpha|Q^2, Q_0^2) \Gamma_{kl}^{\text{QED}}(x_\alpha, x_\beta|Q^2, Q_0^2)$

- **Example:** $Z \to l^+ l^-$ lepton p_T distribution



- Effect of photon PDF from DIS data
  - moderate in the region of the peak
  - rapidly increases away from the peak
- Potentially huge contribution due to lack of constraints from DIS on small-x
  - ruins predictions for high m_Z/p_T^l!
- **Next step:** use W/Z production data to constraint photon PDF $\to$ use for e.g.
  - predictions for jets & Z' production

Reweighting

The mean value of the observable $\mathcal{O}[f]$ taking account o the new data is

$$\langle \mathcal{O} \rangle_{\text{new}} = \frac{1}{N} \sum_{k=1}^{N} w_k \mathcal{O}[f_k]$$

where

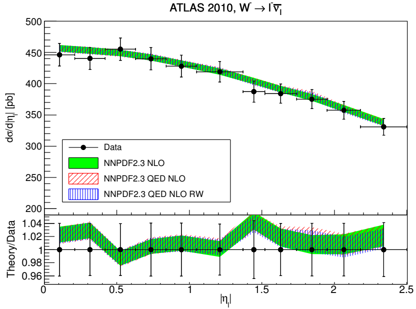
$$w_k = N_\chi \left(\chi^2_k\right)^{\frac{1}{2}(n-1)} e^{-\frac{1}{2}\chi^2_k}$$

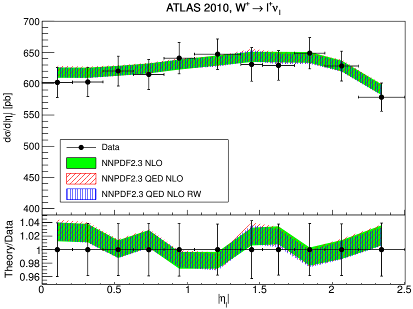
useful equations

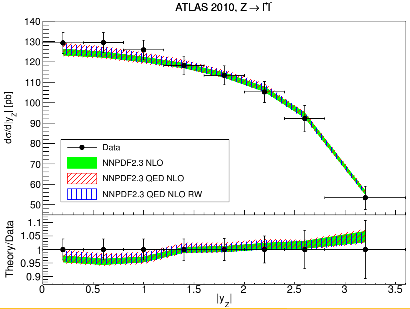
$$P(\alpha) \propto \frac{1}{\alpha} \sum_{k=1}^{N} w_k(\alpha)$$

where $w_k(\alpha)$ are the weights replacing χ^2_k with χ^2_k/α^2.

Reweighting with ATLAS W/Z data (preliminary)





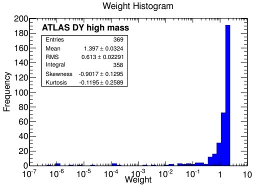


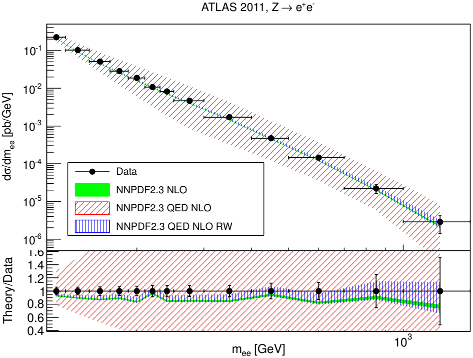
PDF set	χ^2/d.o.f.
NLO pure QCD	1.262
NLO + Photon DIS	1.178
NLO + Photon RW	1.097

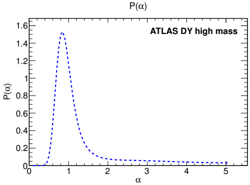
- $Z \to l^+ l^-$ channel is more sensitive to the photon PDF:
  - due to $\gamma\gamma \to l^+ l^-$

Reweighting with ATLAS DY high mass data (prel.)

- For each replica, compute with:
  - **DYNNLO** for the QCD NLO.
  - **HORACE** for the photon induced contribution.
- $N = 500 \rightarrow N_{eff} = 300$







PDF set	χ^2/d.o.f.
NLO pure QCD	0.830
NLO + Photon DIS	5.715
NLO + Photon RW	0.590