1 Feynman rules in momentum space

Throughout this notes we will adopt the following convention for momenta: any derivative acting of a field leads to ip_μ, where the momentum p_μ is flowing *out* the vertex; that is, the wave function of a field is $e^{+ip\cdot x}$ when p is outgoing.

In the SM one has the following Feynman rules in momentum space:

$$\begin{aligned} & Z_\alpha(q) \to W^+_\mu(p_1) W^-_\nu(p_2): \\ & i\epsilon_\alpha(q)\epsilon^*_\mu(p_1)\epsilon^*_\nu(p_2)\left[g\cos\theta_W\right]\left[\eta^{\mu\nu}(p_2-p_1)^\alpha - \eta^{\nu\alpha}(p_2+q)^\mu + \eta^{\mu\alpha}(p_1+q)^\nu\right] \end{aligned} \tag{1}$$

$$\begin{aligned} & W^+_\alpha(q) \to W^+_\mu(p_1) Z_\nu(p_2): \\ & i\epsilon_\alpha(q)\epsilon^*_\mu(p_1)\epsilon^*_\nu(p_2)\left[-g\cos\theta_W\right]\left[\eta^{\mu\nu}(p_2-p_1)^\alpha - \eta^{\nu\alpha}(p_2+q)^\mu + \eta^{\mu\alpha}(p_1+q)^\nu\right] \end{aligned} \tag{2}$$

and also:

$$Z_\alpha(q) \to Z_\mu(p_1) h(p_2): \qquad i\left[2\frac{m_Z^2}{v}\right]\eta^{\mu\alpha}\epsilon_\alpha(q)\epsilon^*_\mu(p_1) \tag{3}$$

$$W^+_\alpha(q) \to W^+_\mu(p_1) h(p_2): \qquad i\left[2\frac{m_W^2}{v}\right]\eta^{\mu\alpha}\epsilon_\alpha(q)\epsilon^*_\mu(p_1)\,, \tag{4}$$

The Feynman rules for the vertices with one ρ and two gauge bosons are ($p_{1,2}$ are flowing out of the vertex, $q \equiv p_1 + p_2$ is flowing into it):

$$\begin{aligned} & \rho^0_\alpha(q) \to W^+_\mu(p_1) W^-_\nu(p_2): \\ & i\epsilon_\alpha(q)\epsilon^*_\mu(p_1)\epsilon^*_\nu(p_2)\left[G^0(q^2)\frac{m_W^2}{p_1\cdot p_2}\right]\left[\eta^{\mu\nu}(p_2-p_1)^\alpha - \eta^{\nu\alpha}(p_2+q)^\mu + \eta^{\mu\alpha}(p_1+q)^\nu\right] \end{aligned} \tag{5}$$

$$\begin{aligned} & \rho^+_\alpha(q) \to W^+_\mu(p_1) Z_\nu(p_2): \\ & i\epsilon_\alpha(q)\epsilon^*_\mu(p_1)\epsilon^*_\nu(p_2)\left[G^\pm(q^2)\frac{m_W m_Z}{p_1\cdot p_2}\right]\left[\eta^{\mu\nu}(p_2-p_1)^\alpha - \eta^{\nu\alpha}(p_2+q)^\mu + \eta^{\mu\alpha}(p_1+q)^\nu\right] \end{aligned} \tag{6}$$

Notice that for an on-shell ρ one has $p_1 \cdot p_2 = (1/2)q^2 = (1/2)m_\rho^2$.

The Feynman rules for the vertices with one ρ, one vector boson and one Higgs boson have the form: ($p_{1,2}$ are flowing out of the vertex, $q \equiv p_1 + p_2$ is flowing into it):

$$\rho^0_\alpha(q) \to Z_\mu(p_1) h(p_2): \qquad i\left[2i\, G^0_H(q^2) m_Z\right]\eta^{\mu\alpha}\epsilon_\alpha(q)\epsilon^*_\mu(p_1) \tag{7}$$

$$\rho^+_\alpha(q) \to W^+_\mu(p_1) h(p_2): \qquad i\left[2i\, G^\pm_H(q^2) m_W\right]\eta^{\mu\alpha}\epsilon_\alpha(q)\epsilon^*_\mu(p_1)\,. \tag{8}$$

In the fermion sector, we assume that the SM fermions couple to the ρ only through its mixing with the elementary $SU(2)_L \times U(1)_Y$ gauge fields, whose Lagrangian reads:

$$\bar{\psi}\, \gamma^\mu \left(g_{el} \frac{\sigma^a}{2} L^a_\mu + g'_{el} Y B_\mu \right) \psi\, , \tag{9}$$

where Y is the hypercharge normalized such that $Y[u_R] = +2/3$. In the mass eigenbasis, the Feynman rules for the vertices with one ρ and two SM fermions are:

$$\rho^+_\mu(q) \to \bar{\psi}_\uparrow(p_1)\psi_\downarrow(p_2) : \quad \bar{u}_\uparrow(p_1)\, V_{CKM} \gamma^\mu \left[\frac{g}{\sqrt{2}} H^\pm_L(q^2) P_L \right] u_\downarrow(p_2) \epsilon_\mu(q) \tag{10}$$

$$\rho^0_\mu(q) \to \bar{\psi}_\uparrow(p_1)\psi_\uparrow(p_2) : \tag{11}$$

$$\bar{u}_\uparrow(p_1)\, \gamma^\mu \left[+\frac{1}{2}(g H^0_L(q^2) - g' H_Y(q^2)) P_L + g' H_Y(q^2) Q[\psi_\uparrow] \right] u_\uparrow(p_2) \epsilon_\mu(q) \tag{12}$$

$$\rho^0_\mu(q) \to \bar{\psi}_\downarrow(p_1)\psi_\downarrow(p_2) : \tag{13}$$

$$\bar{u}_\downarrow(p_1)\, \gamma^\mu \left[-\frac{1}{2}(g H^0_L(q^2) - g' H_Y(q^2)) P_L + g' H_Y(q^2) Q[\psi_\downarrow] \right] u_\downarrow(p_2) \epsilon_\mu(q) \tag{14}$$

where $\psi_\uparrow = \{u, \nu\}$, $\psi_\downarrow = \{d, l\}$ and $P_{L,R} = (1 \pm \gamma_5)/2$.

The on-shell production and decay processes of the ρ are thus controlled by the following parameters: the on-shell values of the form factors $G^{0,\pm}(m^2_\rho)$, $G^{0,\pm}_H(m^2_\rho)$, $H^{0,\pm}_L(m^2_\rho)$, $H_Y(m^2_\rho)$, and the masses of ρ^0 and $\rho^\pm$.

2 Determining the form factors from the $SO(5)/SO(4)$ chiral Lagrangian

We normalize the $SO(5)$ generators T^A ($A = 1{-}10$) so that $\mathrm{Tr}(T^A T^B) = \delta^{AB}$. We distinguish between broken ($SO(5)/SO(4)$) generators $T^{\hat{a}}$ and unbroken ($SO(4)$) generators T^a. The commutation rules can be found in Appendix A of arXiv:1109.1570.

As for the previous case, we follow the CCWZ formalism and use the vector notation where ρ_μ transforms as a gauge field in the adjoint of $SO(4)$. In practice we will consider separately the case of a ρ_L adjoint of $SU(2)_L$ and that of a ρ_R adjoint of $SU(2)_R$. The CCWZ covariant variables are defined by the following equations ($U = \exp(i\Pi(x))$, $\Pi(x) = \sqrt{2} T^{\hat{a}} \pi^{\hat{a}}(x)/f$):

$$-i\, U^\dagger D_\mu U = d^{\hat{a}}_\mu T^{\hat{a}} + E^{L\,a}_\mu T^a_L + E^{R\,a}_\mu T^a_R \equiv d_\mu + E^L_\mu + E^R_\mu\, . \tag{15}$$

The $SO(5)/SO(4)$ chiral Lagrangian then reads, at $O(p^2)$,

$$\mathcal{L}_{(\pi+\rho)} = \frac{f^2}{4} \left(d^{\hat{a}}_\mu \right)^2 - \frac{1}{4 g^2_*} \rho^a_{\mu\nu} \rho^{a\,\mu\nu} + \frac{m^2_*}{2 g^2_*} \left(\rho^a_\mu - E^a_\mu \right)^2 . \tag{16}$$

The index labeling the ρ field runs over the adjoint of $SU(2)_L$ or of $SU(2)_R$. By using the commutation rules for $SO(5)$ it follows ($i = 1, 2, 3$):

$$
\begin{aligned}
d_\mu^{\hat{a}} &= -\frac{\sin\phi}{\sqrt{2}} \left(L_\mu^i \delta^{ai} - B_\mu \delta^{a3} \right) + \sqrt{2}\, \frac{\partial_\mu \pi^{\hat{a}}}{f} + \dots \\
E_\mu^{L\,a} &= \frac{1+\cos\phi}{2}\, L_\mu^a + \frac{1-\cos\phi}{2}\, B_\mu \delta^{a3} + \frac{1}{2f^2} \left[\epsilon^{aij} \pi^i \partial_\mu \pi^j + \delta^{ai} \left(\pi^i \partial_\mu h - h \partial_\mu \pi^i \right) \right] + \dots \\
E_\mu^{R\,a} &= \frac{1-\cos\phi}{2}\, L_\mu^a + \frac{1+\cos\phi}{2}\, B_\mu \delta^{a3} + \frac{1}{2f^2} \left[\epsilon^{aij} \pi^i \partial_\mu \pi^j - \delta^{ai} \left(\pi^i \partial_\mu h - h \partial_\mu \pi^i \right) \right] + \dots
\end{aligned}
\tag{17}
$$

where ϕ is the vacuum misalignment angle, such that $v = f \sin\phi$ and $\xi \equiv (v/f)^2 = \sin^2\phi$.

For $\xi \ll 1$ it is possible to diagonalize the mass matrix in two steps: one can first resolve the composite-elementary mixing before EWSB, and then rotate to find the mass eigenstates after EWSB. Before any rotation, the term of the Lagrangian relevant for the coupling of the ρ to NG bosons reads, for canonically normalized fields,

$$
\mathcal{L}_{\pi+\rho} = -\frac{m_*^2}{2 g_* f^2} \left[\epsilon^{ijk} \pi^j \rho_\mu^k \pi^i \partial^\mu \pm \rho_\mu^k \left(\pi^k \partial_\mu h - h \partial_\mu \pi^k \right) \right] \dots
\tag{18}
$$

where the $+$ ($-$) sign in the second term in squared parenthesis is for a ρ^L (ρ^R).

By performing the elementary-composite rotation one can derive the couplings of the physical ρ to SM fermions and vector bosons. In the case of a ρ_L all three components must be rotated, in an $SU(2)_L$-invariant way, to diagonalize the mass matrix:

$$
\begin{pmatrix} L_\mu^a \\ \rho_\mu^a \end{pmatrix} \to \begin{pmatrix} \cos\theta_L & -\sin\theta_L \\ \sin\theta_L & \cos\theta_L \end{pmatrix} \begin{pmatrix} L_\mu^a \\ \rho_\mu^a \end{pmatrix}, \qquad \tan\theta_L \equiv \frac{g_{el}}{g_*}, \quad g = g_* \sin\theta_L \,.
\tag{19}
$$

The masses of the heavy mass eigenstates and the strength of the $\rho^+\rho^-\rho^0$ coupling are, before EWSB,

$$
m_{\rho^\pm} = m_{\rho^0} = m_\rho = \frac{m_*}{\cos\theta_L}\,, \qquad g_\rho = g_* \frac{\cos 2\theta_L}{\cos\theta_L} = 2g \cot 2\theta_L\,,
\tag{20}
$$

hence

$$
a_\rho \equiv \frac{m_{\rho^+}}{g_\rho f} = \frac{m_*}{g_* f} \frac{1}{\cos 2\theta_L} \equiv a_* \frac{1}{\cos 2\theta_L} = a_* \sqrt{1 + 4\frac{g^2}{g_\rho^2}}\,.
\tag{21}
$$

The form factors are:

$$G^0(q^2) = G^\pm(q^2) = \frac{m_\rho^2}{2f^2}\frac{\cos 2\theta_L}{g_\rho} = \frac{m_\rho^2}{2g_\rho f^2}\frac{1}{\sqrt{1+4\frac{g^2}{g_\rho^2}}} \tag{22}$$

$$G_H^0(q^2) = G_H^\pm(q^2) = \frac{m_\rho^2}{2f^2}\frac{\cos 2\theta_L}{g_\rho} = \frac{m_\rho^2}{2g_\rho f^2}\frac{1}{\sqrt{1+4\frac{g^2}{g_\rho^2}}} \tag{23}$$

$$H_L^0(q^2) = H_L^\pm(q^2) = -\tan\theta_L = \frac{1}{2}\left(\frac{g_\rho}{g} - \sqrt{4+\frac{g_\rho^2}{g^2}}\right) \tag{24}$$

$$H_Y(q^2) = 0\,, \tag{25}$$

In the case of a ρ_R, instead, only the neutral component undergoes the elementary-composite mixing:

$$\begin{pmatrix} B_\mu \\ \rho_\mu^3 \end{pmatrix} \to \begin{pmatrix} \cos\theta_R & -\sin\theta_R \\ \sin\theta_R & \cos\theta_R \end{pmatrix}\begin{pmatrix} B_\mu \\ \rho_\mu^0 \end{pmatrix}\,, \qquad \tan\theta_R \equiv \frac{g'_{el}}{g_*}\,, \quad g' = g_*\sin\theta_R\,. \tag{26}$$

so that the physical masses and $\rho^+\rho^-\rho^0$ couplings strength read:

$$m_{\rho^\pm} = m_*\,, \qquad m_{\rho^0} = \frac{m_*}{\cos\theta_R} = m_{\rho^\pm}\sqrt{1+\frac{g'^2}{g_\rho^2}}\,, \qquad g_\rho = g_*\cos\theta_R = g'\cot\theta_R\,, \tag{27}$$

hence

$$a_\rho \equiv \frac{m_{\rho^+}}{g_\rho f} = \frac{m_*}{g_* f}\frac{1}{\cos\theta_R} \equiv a_*\frac{1}{\cos\theta_R} = a_*\sqrt{1+\frac{g'^2}{g_\rho^2}}\,. \tag{28}$$

The form factors are:

$$G^0(q^2) = \frac{m_{\rho^0}^2\cos^2\theta_R}{2g_\rho f^2} = \frac{m_{\rho^0}^2}{2g_\rho f^2}\frac{1}{1+\frac{g'^2}{g_\rho^2}}, \qquad G^\pm(q^2) = \frac{m_{\rho^\pm}^2\cos\theta_R}{2g_\rho f^2} = \frac{m_{\rho^\pm}^2}{2g_\rho f^2}\frac{1}{\sqrt{1+\frac{g'^2}{g_\rho^2}}} \tag{29}$$

$$G_H^0(q^2) = -\frac{m_{\rho^0}^2\cos^2\theta_R}{2g_\rho f^2} = -\frac{m_{\rho^0}^2}{2g_\rho f^2}\frac{1}{1+\frac{g'^2}{g_\rho^2}}, \quad G_H^\pm(q^2) = -\frac{m_{\rho^\pm}^2\cos\theta_R}{2g_\rho f^2} = -\frac{m_{\rho^\pm}^2}{2g_\rho f^2}\frac{1}{\sqrt{1+\frac{g'^2}{g_\rho^2}}} \tag{30}$$

$$H_L^0(q^2) = H_L^\pm(q^2) = 0 \tag{31}$$

$$H_Y(q^2) = -\tan\theta_R = -\frac{g'}{g_\rho}\,. \tag{32}$$